PHYS 1114, Lecture 10, February 8

Contents:

1° Example of projectile motion: Man shooting a gun firing a bullet horizontally.
2° Example of projectile motion: Man shooting an arrow at a monkey in a tree. This is a worked out numerical example related to an earlier movie shown.
3° Concepts of force and net force are introduced.
4° Different types of forces are mentioned.
5° Free-body diagrams are introduced with the example of a man pushing a piano.
6° Concept of equilibrium (statics) is defined. Statics is an elaborate course in engineering, required for architecture majors.
7° Movie: Newton’s 1st Law: Rest Inertia of a Massive Ball
   a. A heavy metal ball is hanging suspended by a thread. A similar thread is attached to the bottom of the ball. A force will be applied to this second thread by pulling the bottom end of it down.
   b. If one pulls the bottom thread slowly the top thread breaks. The top thread experiences the tension due to the weight of the ball in addition to the force exerted by pulling the bottom thread.
   c. If one pulls the bottom thread quickly the bottom thread breaks. The massive ball has inertia resisting a sudden change of its velocity. [The resulting extra force on the bottom string is quantified by Newton’s Second Law: mass times acceleration of the ball.]
**Projectile Motion Example 1:** A man stands on a wall and shoots a gun. The bullet leaves the gun horizontally 9.80 m above the ground below and with a speed of 44.1 m/s. How far will the bullet travel horizontally before hitting the ground? How long will it be in the air?

a. Draw a picture on the left marking all variables.
b. Make a table on the right of these variables.
c. Choose the equations to be solved.
d. Solve the problem.

\[
\begin{array}{l}
\text{variable} & \text{value} \\
\hline
x_o & 0 \\
x & ? \\
y_o & 9.80 \text{ m} \\
y & 0 \\
v_x & 44.1 \text{ m/s} \\
v_yo & 0 \\
v_y & ? \\
a_y & -9.80 \text{ m/s}^2 \\
t & ? \\
\end{array}
\]

\[
x, t? \quad \rightarrow \quad x = x_o + v_{xo}t
\]

\[
t? \quad \rightarrow \quad y = y_o + v_{yo}t + \frac{1}{2}a_yt^2
\]

\[
v_y, t? \quad \rightarrow \quad v_y = v_{yo} + a_yt
\]

\[
v_y? \quad \rightarrow \quad v_y^2 = v_{yo}^2 + 2a_y(y - y_o)
\]
We see that we can solve the second and fourth equations, but we cannot solve the first and third ones without solving one of the other two.

Start with the second one and then, with the solution for $t$, solve the first and third ones:

\[0 = 9.80 \, \text{m} + 0 - \frac{1}{2} 9.80 \, \text{m/s}^2 \, t^2 \quad \implies \quad t = \sqrt{\frac{2 \times 9.80 \, \text{m}}{9.80 \, \text{m/s}^2}}\]

\[t = 1.4142 \, \text{s} \quad \implies \quad t = 1.41 \, \text{s}\]

and, as $v_x$ is constant,

\[x = 0 + 44.1 \, \text{m/s} \times 1.4142 \, \text{s} \quad \implies \quad x = 62.4 \, \text{m} \, .\]

As an extra, we can also find

\[v_y = v_{yo} - gt = 0 - (9.80 \, \text{m/s}^2)(1.4142 \, \text{s})\]

\[= 13.9 \, \text{m/s} \, .\]

(We have used extra digits in intermediate steps, not to increase rounding errors. The final answers are to three digits.)
Projectile Motion Example 2: An archer shoots an arrow with an initial speed of 180 km per hour aimed straight at a monkey hanging on a tree branch. The archer stands 40.0 m from the tree and the monkey is 11.5 m above the ground; the arrow is released 1.5 m above the ground. The monkey drops from the branch as soon as the arrow is released. (Ignore air resistance.)

a) Make a sketch of the situation.

![Sketch of the situation]

b) What is the initial speed of the arrow in m/s? What angle $\theta$ does the initial velocity make with the horizontal plane? Calculate the initial horizontal and vertical components $v_{xo}$ and $v_{yo}$ of the initial velocity.

$$v_o = \frac{180 \text{ km/hour} \times \frac{1000 \text{ m/km}}{3600 \text{ s/hour}}}{50 \text{ m/s}}$$

$$\theta = \tan^{-1} \left( \frac{11.5 \text{ m} - 1.5 \text{ m}}{40 \text{ m}} \right) = \tan^{-1} \frac{1}{4} = 0.245 \text{ rad} = 14.0^\circ$$

$$v_{xo} = v_o \cos \theta = 48.5 \text{ m/s}, \quad v_{yo} = v_o \sin \theta = 12.1 \text{ m/s}$$
c) Which equations can be used for the horizontal part of the motion of the arrow? How long does it take for the arrow to reach the monkey?

\[ v_x = v_{xo}, \quad x = x_0 + v_{xo} t \]

\[ t = \frac{x}{v_{xo}} = \frac{40.0 \text{ m}}{48.5 \text{ m/s}} = 0.825 \text{ s} \]

d) Which equations can be used for the vertical part of the motions?

Using \( a_y = -g \),

\[ v_y = v_{yo} - gt, \quad y = y_0 + v_{yo} t - \frac{1}{2} gt^2 \]

However, the third equation \( v_y^2 = v_{yo}^2 - 2g(y - y_0) \) has two unknowns, \( y \) and \( v_y \).

e) Will the monkey be hit? How much will the monkey have dropped at the time the arrow reaches the tree?

[YES]

\[ d = \frac{1}{2} gt^2 = \frac{1}{2} \times (9.80 \text{ m/s}^2) \times (0.825 \text{ s})^2 = 3.33 \text{ m} \]

Indeed, for the monkey \( v_{yo} = 0 \), so \( y - y_0 = -d = -\frac{1}{2} gt^2 \).

Compare this with \( y - (y_0 + v_{yo} t) \), the deviation from the straight line the arrow originally followed. This is the same \(-d = -\frac{1}{2} gt^2\).
Forces

A force $\mathbf{F} = \vec{F}$:

- is a push or a pull
- acts on an object
- requires an agent
- is a vector
- is a contact force or a long-range force

Net force is the resultant of all forces acting on the object:

$$\vec{F}_{\text{net}} = \sum_{i=1}^{N} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N$$

Net force = vector sum

Force is typically measured with a spring gauge. Two different kinds were shown, one with a linear scale and one with a dial scale.
Some types of forces

- weight (gravity)
- spring force
- tension
- normal force
- friction \{ static, kinetic \}
- drag (air friction)
- thrust (of jet or rocket)
- electro-magnetic forces
- weak and strong forces
- push or pull by a person

Gravitational, electromagnetic, weak, and strong forces are fundamental forces of nature. Gravity will be discussed in more detail in this course, the other three appear in Physics 1214.
Free-Body Diagram

Example: A man pushing a piano. Besides the push there are three other forces on the piano, i.e. weight, normal force preventing the piano to go through the floor, and friction.

Here:  \( \vec{N} = \vec{w}, \)  (or  \( \vec{N} = \vec{w} \)).

In a free-body diagram we often replace the object by a point and we indicate all forces acting on it.  \( \implies \) Point mass picture:

Usually we also draw two coordinate axes, placing the point mass in the origin and using labels \( x \) and \( y \) to indicate the positive \( x \)- and \( y \)-directions.

\[
\text{Equilibrium} = \text{Statics}: \sum \vec{F} = 0
\]