Contents:

1° Banked curve example worked out.

2° Tension in ball-on-string circular motion.

3° Fictitious forces in rotating frame.

4° Newton’s Universal Gravity.

5° Kepler’s Laws of Planetary Motion

6° *Simple Experiment:* A ball on a string is made to make a horizontal circular motion, as the hand holding the other end of the string moves in a small circle. At low speed the string is almost vertical. As the speed is increased, the circle widens and moves up. At high speed the string is nearly horizontal and the hand feels a much increased tension.

    Next make the ball move in a vertical circle. Now there is a minimum speed needed to make the full circle, without the ball falling out of it before reaching the top. This minimum speed is again the critical speed $v_{\text{crit}}$ seen earlier in the loop-the-loop movie.
Turning banked corner:

No friction
\(F_s = 0\)
\(r = 100\, \text{m}\)
\(\theta = 20^\circ\)
\(v_\theta = \?)

\[\begin{align*}
\sum F_r &= F_N \sin \theta = F_c \\
\sum F_z &= F_N \cos \theta - w = 0
\end{align*}\]

\[\Rightarrow \tan \theta = \frac{F_N \sin \theta}{F_N \cos \theta} = \frac{F_c}{mg} = \frac{\eta h v^2}{\eta h g} = \frac{v^2}{rg} \Rightarrow v^2 = rg \tan \theta\]

\[\Rightarrow v = \sqrt{rg \tan \theta}\]

\(\theta = 20^\circ, \quad r = 100\, \text{m}, \quad g = 9.80\, \text{m/s}^2, \quad \tan \theta = 0.36397 \cdots, \quad v = \sqrt{356.69 \cdots \text{m}^2/\text{s}^2} = 18.886\, \text{m/s} = [19\, \text{m/s}] = 68\, \text{km/h.}\)

(Book: \(\theta = 65.0^\circ, \tan \theta = 2.14, \quad v = 45.8\, \text{m/s} = 165\, \text{km/h.}\))
Ball on rope: Vertical circular motion

Resultant force on top provides centripetal force $F_c$:

$$F_c = \frac{mv^2}{r} = mg + T \geq mg,$$

$$v^2 \geq rg, \text{ or } v \geq v_c = \sqrt{rg}.$$ 

Ball on rope: Horizontal circular motion

$$mg = T \sin \theta, \quad \{$$

$$F_c = T \cos \theta, \quad \}$$

divide

$$\frac{mg}{F_c} = \sin \theta \cos \theta = \tan \theta \quad \Rightarrow$$

$$\tan \theta = \frac{mg}{\frac{mg}{v^2}} = \frac{rg}{v^2} \quad \Rightarrow \quad \theta = \arctan \left( \frac{rg}{v^2} \right) = \tan^{-1} \left( \frac{rg}{v^2} \right)$$

and

$$T = \frac{mg}{\sin \theta} = \frac{mg}{\sin[\arctan(rg/v^2)]}$$
Rotating Frame is **not** inertial frame

(Example: merry-go-around)

Fictitious forces:  (not real forces)
- Centrifugal force
- Coriolis force

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**Centrifugal force:**

Observer sees object rotate  Observer sees object at rest

In both cases centripetal force $F_c$ is a real physical force.

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**Coriolis force:** Not part of this course. This force, for example, causes a hurricane to rotate counterclockwise on the Northern hemisphere.
Newton’s Universal Law of Gravity

\[ F = \frac{GmM}{r^2}, \quad G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

In 1798 Cavendish already found \( G \) to better than 1\% (see figure 6.28 in the book). A somewhat conceptually easier way is moving two masses on strings together as follows:

\[ \tan \theta = \frac{F_{\text{Newton}}}{mg} = \frac{Gm^2}{d^2} = \frac{Gm}{gd^2} \]

Knowing \( \theta, d, m \) and \( g \) one can find \( G \).
Planetary Motion

Kepler’s 1st law: orbit = ellipse (from Tycho Brahe’s observations)

Kepler’s 2nd law: In equal times equal areas (perks) are swept.

[Dutch: perkenwet (law of the perks)]

Kepler’s 3rd law: Relates period $T$ and long axis of the ellipse.

For circle of radius $r$, $v = \frac{2\pi r}{T}$.

Gravity = centripetal force:

$$\frac{G M m}{r^2} = \frac{\eta v^2}{r} \implies GM = v^2 r = \left(\frac{2\pi r}{T}\right)^2 r = \frac{4\pi^2 r^3}{T^2} \implies$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} = \text{const} \implies r^3 \propto T^2.$$