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7° Movie: *Newton’s 3rd Law: Reaction Cart/Projected Ball Bearings*
   a. This is a mechanical model of a jet engine.
   b. Ball bearings are loaded on lightly sloped guide rails on top of a cart. The rails connect to a nearly vertical pipe behind the cart, that turns horizontal again near the floor (the chute).
   c. The Third Law is demonstrated two ways:
      1°) As the balls roll down the slope the cart moves in the opposite direction, keeping the center of mass of cart plus ball bearings in the same horizontal position. (This involves concepts treated in later lectures.)
      2°) When the balls leave the chute, each ball pushes the cart while the cart pushes each ball. When many balls exit the chute the cart is seen to speed up, just like a jet is pushed forward by each molecule leaving its jet engines.
8° Motion of Center of Mass:
System with Internally Moving Components

a. A glider with a pendulum attached is placed on an airtrack. The center of mass of the glider-pendulum system is marked with a small white circle on the pendulum arm. With friction the glider cannot move and the pendulum swings back and forth. Once the airflow is turned on, the glider can move frictionless and is seen to move back and forth as the pendulum swings. However, the center of mass does not move (or, more precisely, goes slightly up and down).

b. If the glider is pushed forward without the pendulum swinging, it moves uniformly towards the end of the airtrack. However, if it is moved with the pendulum swinging, it makes a jittery motion and only the mark of the center of mass is seen to move uniformly.
9° Experiment: Newton’s Cradle.

A multiple pendulum consisting of an array of identical metal balls suspended by strings as shown in the first figure below is used.

Releasing one ball as in the middle figure a sequence of collisions occurs whereby each ball exchanges momentum with the next ball. The last ball swings out as in the third figure. This process repeats over and over, with either the left or the right ball swinging out.

Taking two balls out instead of one, then after the chain of collisions two balls swing out at the other side. Taking three balls, then three, etc.
Momentum and Impulse

Linear momentum

\[ \vec{p} = m\vec{v} \]

Total mechanical (linear) momentum

\[ \vec{p}_{\text{tot,mech}} = \sum m\vec{v} \]

Total energy conserved
\[ \iff \text{physics the same at all times in our age} \]

Total linear momentum conserved
\[ \iff \text{physics the same everywhere in the universe} \]

Total angular momentum conserved (later lecture)
\[ \iff \text{physics the same at all angles} \]

\[ \text{Impulse} = \Delta\vec{p} = \overrightarrow{F}_{\text{net}} \Delta t \]
\[ \uparrow \]
(average net force)
Crowd control

The water flows out of the hose at rate \( \begin{align*} &30 \text{ kg/s at} \\
v &= 20 \text{ m/s} \end{align*} \)

\[
\Delta t = 1 \text{ s}, \quad m = 30 \text{ kg} \quad v = 20 \text{ m/s} \quad p = mv = 600 \text{ kg} \cdot \text{m/s} = 600 \text{ N} \cdot \text{s}
\]

Assume that the man on the right does not move.
We can then calculate the average force he exerts on the beam of water:

\[
p_o = 600 \text{ N} \cdot \text{s}, \quad p_f = 0
\]

\[
\overrightarrow{F} = \frac{p_f - p_o}{\Delta t} = \frac{0 - 600 \text{ kN} \cdot \text{s}}{1 \text{ s}} = -600 \text{ N}
\]
Collisions

Before

\[ m_1 \quad v_1 = v_{1o} \quad m_2 \quad v_2 = v_{2o} \]

After

\[ v_1' = v_{1f} \quad m_1 \quad m_2 \quad v_2' = v_{2f} \]

\[ \Delta \vec{p}_1 = m_1 \vec{v}_1' - m_1 \vec{v}_1 = \bar{F}_{12} \Delta t \]

average force by 2 on 1
\[ \Delta t = \text{duration of collision} \]

\[ \Delta \vec{p}_2 = m_2 \vec{v}_2' - m_2 \vec{v}_2 = \bar{F}_{21} \Delta t = -\bar{F}_{12} \Delta t \]

average force on 2 by 1
\[ \text{Newton’s 3rd law} \]

\[ \Delta \vec{p}_1 + \Delta \vec{p}_2 = 0 \quad \implies \]

\[ \vec{p}_{1f} - \vec{p}_{1o} + \vec{p}_{2f} - \vec{p}_{2o} = 0 \quad \implies \]

\[ \vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1o} + \vec{p}_{2o} \]

Momentum is conserved in collisions.
\[ m_1 \ddot{v}_1 + m_2 \ddot{v}_2 = m_1 \ddot{v}_1' + m_2 \ddot{v}_2' \]

Isolated system of \( N \) masses:

\[ \vec{F}_{ij} = -\vec{F}_{ji} \]

In each separate collision \( \Delta \vec{p}_{\text{total}} = 0 \) implying

\[ m_1 \ddot{v}_1 + m_2 \ddot{v}_2 + \cdots + m_N \ddot{v}_N = m_1 \ddot{v}_1' + m_2 \ddot{v}_2' + \cdots + m_N \ddot{v}_N' \]

\[ \Delta (m_1 \ddot{v}_1 + m_2 \ddot{v}_2 + \cdots + m_N \ddot{v}_N) = 0 \]

\[ \boxed{\Delta \vec{p}_{\text{tot}} = 0} \]

Center of mass:

\[ \vec{r}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots + m_N \vec{r}_N}{m_1 + m_2 + \cdots + m_N} \]

(weighted average position)

\[ \frac{\Delta \vec{r}_{CM}}{\Delta t} = \frac{m_1 \frac{\Delta \vec{r}_1}{\Delta t} + m_2 \frac{\Delta \vec{r}_2}{\Delta t} + \cdots + m_N \frac{\Delta \vec{r}_N}{\Delta t}}{m_1 + m_2 + \cdots + m_N} \]

\[ \vec{v}_{CM} = \frac{m_1 \ddot{v}_1 + m_2 \ddot{v}_2 + \cdots + m_N \ddot{v}_N} {m_1 + m_2 + \cdots + m_N} = \frac{\vec{p}_{\text{tot}}}{m_{\text{tot}}} \]
\[ \Delta \vec{p}_{\text{tot}} = 0 \implies \vec{v}_{\text{CM}} = \text{constant in isolated systems} \]

This is the reason of treating extended objects as “point masses.”

Types of collisions:

1) Elastic collision

\[ \vec{p}_{\text{tot}}, \quad E_{\text{tot,mech}} \quad \text{conserved} \]

2) Completely inelastic collision

\[ \vec{v}_1' = \vec{v}_2' \]

3) Inelastic collision (but not completely)

[Hard]

(We shall not consider this intermediate case in this course.)
Completely inelastic collision

\[
\begin{align*}
\begin{cases}
m_1 \ddot{v}_1 + m_2 \ddot{v}_2 &= m_1 \ddot{v}_1' + m_2 \ddot{v}_2' \\
&\text{before} \quad \text{after}
\end{cases}
\end{align*}
\]
\[
\ddot{v}_1' = \ddot{v}_2' = \ddot{v}'
\]
\[
m_1 \ddot{v}_1 + m_2 \ddot{v}_2 = (m_1 + m_2) \ddot{v}'
\]
\[
\ddot{v}' = \frac{m_1 \ddot{v}_1 + m_2 \ddot{v}_2}{m_1 + m_2}
\]

Shoot gun at wood block

\[
\begin{align*}
m, \; v_1 &\quad M, \; v_2 = 0 \\
\text{frictionless track} &
\end{align*}
\]
\[
v_1' = v_2' = v' = ?
\]
\[
m v_1 = (m + M) v'
\]
\[
v_1 = \frac{m + M}{m} v', \quad v' = \frac{m}{m + M} v_1
\]