Diagnostic Mathematics Proficiency Test (White)

(Not counted for grade. Please, write your intermediate steps also.)

1. Calculate in scientific notation, only listing the significant figures in the final answer:
   \((2.0 \times 10^6)(3.5 \times 10^3) + 7.1 \times 10^4 = \)

2. Solve \(2x + 2 = x + 7\).

3. Solve \(\begin{cases} 
   x + y = 1, \\
   2x + 2y = 2, \\
   3x + 5y = 4.
\end{cases}\)

4. Solve \(3x^2 + 7x + 2 = 0\).

5. \((3 \times x^2)(5 \times x^3) + (x^{\frac{1}{2}})^{10} = \)

6. \(\sin^2 89^\circ + \sin^2 1^\circ = \)

7. Given \(\theta = 0.760\) radians, \(\sin \theta = \)

8. Given \(\theta = 76.5^\circ\), \(\tan \theta = \)

9. Given \(\cot \theta \equiv 1/\tan \theta = 0.760\), \(\theta = \)

10. Given this triangle with \(a=1.60, b=2.00, \alpha = 49^\circ\), in some units, calculate \(\beta\) and \(\gamma\).

\[\beta = \]

\[\gamma = \]
Diagnostic Mathematics Proficiency Test (Yellow)
(Not counted for grade. Please, write your intermediate steps also.)

1. Calculate in scientific notation, only listing the significant figures in the final answer:
\[(3.0 \times 10^3)(2.5 \times 10^5) + 7.1 \times 10^4 =\]

2. Solve \[2x + 4 = x + 7.\]

3. Solve \[
\begin{cases}
  x + y = 1, \\
  2x + 2y = 2, \\
  5x + 3y = 4.
\end{cases}
\]

4. Solve \[2x^2 + 7x + 3 = 0.\]

5. \[(4 \times x^4)(3 \times x^4) + \left(x^{\frac{1}{2}}\right)^{16} =\]

6. \[\sin^2 61^\circ + \sin^2 29^\circ =\]

7. Given \(\theta = 0.640\) radians, \(\sin \theta =\)

8. Given \(\theta = 64.5^\circ\), \(\tan \theta =\)

9. Given \(\cot \theta \equiv 1/ \tan \theta = 0.640\), \(\theta =\)

10. Given this triangle with \(a=2.40\), \(b=3.00\), \(\alpha = 49^\circ\),
in some units, calculate \(\beta\) and \(\gamma\).

\[\beta =\]

\[\gamma =\]
Diagnostic Mathematics Proficiency Test (White)

(Answers with additional comments.)

1. Calculate in scientific notation, only listing the significant figures in the final answer:
   \[(2.0 \times 10^6)(3.5 \times 10^3) + 7.1 \times 10^4 = 7.0 \times 10^9\] (Last term is insignificant.)

2. Solve \[2x + 2 = x + 7. \implies 2x - x = 7 - 2 \implies x = 5.\]

3. Solve \[
\begin{align*}
x + y &= 1, \\
2x + 2y &= 2, \\
3x + 5y &= 4.
\end{align*}
\]
   \[
\begin{align*}
y &= 1 - x, \\
\text{Same,} \\
3x + 5y &= 4.
\end{align*}
\]
   \[
\begin{align*}
\implies 3x + 5 - 5x &= 4 \\
5 - 2x &= 4 \\
2x &= 1 \\
x &= \frac{1}{2}, \quad y = \frac{1}{2}.
\end{align*}
\]

4. Solve \[3x^2 + 7x + 2 = 0. \implies x = \frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times 2}}{2 \times 3} \implies x = \frac{-7 \pm \sqrt{25}}{6} \implies x = \frac{-7 \pm 5}{6}
\]
   \[
\begin{align*}
\implies x &= \frac{-12}{6} \quad \text{or} \quad \frac{-2}{6} \implies x = -2 \quad \text{or} \quad -\frac{1}{3}.
\end{align*}
\]

5. \[(3 \times x^2)(5 \times x^3) + \left(x^{\frac{4}{2}}\right)^{10} = 15x^5 + x^5 = 16x^5.\]

6. \[\sin^2 89^\circ + \sin^2 1^\circ = 1, \quad \text{as} \quad \sin^2 \theta + \cos^2 \theta = 1.\]

7. Given \(\theta = 0.760 \text{ radians}, \quad \sin \theta = 0.689.\)

8. Given \(\theta = 76^\circ 30' = 76.5^\circ, \quad \tan \theta = 4.17.\)

9. Given \(\cot \theta = 0.760, \quad \theta = 0.921 \text{ rad (mod } \pi) = 52.77^\circ \text{ (mod } 180^\circ),\) (as \(\cot \theta = 1/\tan \theta).\)

10. Given this triangle with \(a=1.60, b=2.00, \alpha = 49^\circ,\) in some units, calculate \(\beta\) and \(\gamma.\) (\(\gamma = 180^\circ - \alpha - \beta.\))

   \[
\frac{\sin \beta}{b} = \frac{\sin \alpha}{a} \implies \sin \beta = \frac{b}{a} \sin \alpha = \frac{2.00}{1.60} \times 0.755
\]
   \[\implies \sin \beta = 0.943 \implies \beta = 1.233 \text{ rad} = 70.63^\circ\]

   (or \(\beta = 1.909 \text{ rad} = 109.37^\circ\)), and

   \[\gamma = 1.054 \text{ rad} = 60.37^\circ\] (or \(0.378 \text{ rad} = 21.63^\circ\))

   \[\text{Extra:} \quad a^2 = b^2 + c^2 - 2bc \cos \alpha \implies c^2 - 2bc \cos \alpha + b^2 - a^2 = 0 \implies c = \frac{2b \cos \alpha \pm \sqrt{(2b \cos \alpha)^2 - 4(b^2 - a^2)}}{2} \implies c = b \cos \alpha \pm \sqrt{a^2 - b^2 \sin^2 \alpha} \quad \uparrow\]

\[\uparrow\] Noting \(\gamma = 180^\circ - \alpha - \beta = \pi \text{ rad} - \alpha - \beta,\) we can also use \(c = a \sin \gamma / \sin \alpha = b \sin \gamma / \sin \beta.\)

The auxiliary line is of length \(b \sin \alpha = a \sin \beta,\) and \(c = b \cos \alpha + a \cos \beta\) gives a third way to calculate \(c.\) The boxed answers follow the figure with \(\cos \beta > 0, \beta < 90^\circ = \frac{1}{2} \pi \text{ rad.}\)
Diagnostic Mathematics Proficiency Test (Yellow)
(Answers with additional comments.)

1. Calculate in scientific notation, only listing the significant figures in the final answer:
   \[ (3.0 \times 10^3)(2.5 \times 10^5) + 7.1 \times 10^4 = 7.5 \times 10^8 \]  
   (Last term is insignificant.)

2. Solve \( 2x + 4 = x + 7 \).  
   \( \implies 2x - x = 7 - 4 \implies x = 3 \).

3. Solve \( \begin{align*} 
   x + y &= 1, \\
   2x + 2y &= 2, \\
   5x + 3y &= 4.
\end{align*} \)
   \( \implies \begin{align*} 
   y &= 1 - x, \\
   5x + 3y &= 4.
\end{align*} \)
   \( \implies 5x + 3 - 3x = 4 \implies 5x - 3x = 4 - 3 \implies 2x = 1 \implies x = \frac{1}{2}, \ y = \frac{1}{2} \).

4. Solve \( 2x^2 + 7x + 3 = 0 \).  
   \( \implies x = \frac{-7 \pm \sqrt{7^2 - 4 \times 2 \times 3}}{2 \times 3} \)
   \( \implies x = \frac{-7 \pm \sqrt{49 - 24}}{4} \implies x = \frac{-7 \pm \sqrt{25}}{4} \implies x = \frac{-7 \pm 5}{4} \)
   \( \implies x = \frac{-2}{4} \) or \( \frac{-12}{4} \) \( \implies x = -\frac{1}{2} \) or \( x = -3 \).

5. \( (4 \times x^4)(3 \times x^4) + \left(x^\frac{1}{2}\right)^{16} = 12x^8 + x^8 = 13x^8 \).

6. \( \sin^2 61^\circ + \sin^2 29^\circ = 1, \quad \text{as} \quad \sin^2 \theta + \cos^2 \theta = 1 \).

7. Given \( \theta = 0.640 \) radians, \( \sin \theta = 0.597 \).

8. Given \( \theta = 64^\circ 30' = 64.5^\circ, \tan \theta = 2.10 \).

9. Given \( \cot \theta = 0.640, \theta = 1.001 \text{ rad (mod } \pi\text{)} = 57.38^\circ \text{ (mod } 180^\circ\text{)}, \) (as \( \cot \theta = 1/\tan \theta \)).

10. Given this triangle with \( a=2.40, b=3.00, \alpha = 49^\circ \), in some units, calculate \( \beta \) and \( \gamma \). (\( \gamma = \pi - \alpha - \beta \).)

   \[ \frac{\sin \beta}{b} = \frac{\sin \alpha}{a} \implies \sin \beta = \frac{b}{a} \sin \alpha = \frac{3.00}{2.40} \times 0.755 \]
   \( \implies \sin \beta = 0.943 \implies \beta = 1.233 \text{ rad} = 70.63^\circ \)

   \( \text{(or } \beta = 1.909 \text{ rad} = 109.37^\circ\text{), and} \)

   \[ \gamma = 1.054 \text{ rad} = 60.37^\circ \]  
   \( \text{(or } 0.378 \text{ rad} = 21.63^\circ) \)

Extra: \( a^2 = b^2 + c^2 - 2bc \cos \alpha \implies c^2 - 2bc \cos \alpha + b^2 - a^2 = 0 \implies \)
\( c = b \cos \alpha \pm \sqrt{(2b \cos \alpha)^2 - 4(b^2 - a^2)} \)
\( \implies c = b \cos \alpha \pm \sqrt{a^2 - b^2 \sin^2 \alpha} \)

\[ \uparrow \text{ Noting } \gamma = 180^\circ - \alpha - \beta = \pi \text{ rad} - \alpha - \beta, \text{ we can also use } c = a \sin \gamma / \sin \alpha = b \sin \gamma / \sin \beta. \]

The auxiliary line is of length \( b \sin \alpha = a \sin \beta \), and \( c = b \cos \alpha + a \cos \beta \) gives a third way to calculate \( c \). The boxed answers follow the figure with \( \cos \beta > 0, \beta < 90^\circ = \frac{1}{2} \pi \text{ rad.} \)