Department of Physics

Preliminary Examination

Classical Mechanics

Monday, January 5, 1998

9:00 am - noon

Instructions:

1. Write your answer to each question on a separate sheet of paper. If more than one sheet is needed staple all of the pages corresponding to a single question together in the correct order. But, do not staple all of the problems together. This exam has 5 questions.

2. Be sure to write your identification number (not your name) and the problem number on each problem sheet.

3. The time allowed for the exam is three hours. All questions carry the same amount of credit. Consequently, you should manage your time carefully.

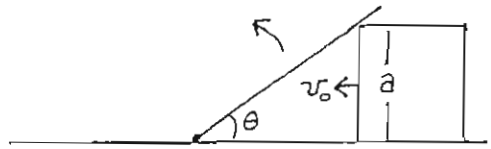
4. Several of the questions have more than one part. It may not be necessary to successfully complete one part to do the other parts.

5. Grades on the exam will be determined, in part, by such things as the clarity and organization of the response. Thus, it is a good idea to use short written statements between the lines of a derivation, etc. Be sure to substantiate any answer by calculations or arguments, as appropriate. Be complete, explicit, and brief.

6. The use of electronic calculators is permitted. However, obtaining programmed information from programmable calculators, using tables, books, paper, or other reference material that is not provided by the exam committee is strictly prohibited. The OSU regulations of academic dishonesty will be followed.

7. All questions carry the same weight.

1 As shown in the diagram, a rod fixed at one end and allowed to rotate rests against a box having height "a". The box is then moved at a constant speed v_0 thereby imparting an angular acceleration to the rod. Find the instantaneous angular velocity ω and the instantaneous angular acceleration α of the rod.



2 A one dimensional diatomic molecule is composed of atoms with masses m_1 and m_2 The atoms interact by way of the Lennard-Jones potential energy function

$$V(x) = \epsilon \left\{ -2\left[\frac{\sigma}{x}\right]^6 + \left[\frac{\sigma}{x}\right]^{12} \right\}$$

where $x = |x_2 - x_1|$ is the interatomic distance and ϵ and σ are two parameters which are specified empirically for the molecule.

1) Find the "vibrational (angular) frequency" ω_0 for small amplitude oscillations of the diatomic molecule.

Note a complete solution of the problem should contain such notions as

i) the separation of center of mass and relative coordinates;
ii) the "reduced mass" of the system;
iii) the "equilibrium separation" of the two atoms;
iv) the "harmonic oscillator" approximation for the relative motion of the atoms.
v) the solution of the equation of motion.

2) For carbon monoxide, $C^{12}O^{16}$, the atomic masses and the Lennard-Jones parameters are

$$m_C = 2.00 \times 10^{-26}\ \text{kg} \quad \text{and} \quad m_O = 2.67 \times 10^{-26}\ \text{kg}$$

$$\epsilon \simeq 1.38 \times 10^{-21}\ \text{J} \qquad \sigma = 4.0 \times 10^{-10}\ \text{m}$$

In what region of the electromagnetic spectrum would you expect to find absorption?

3 CONSTANTS OF MOTION AND SYMMETRY. Write down all conserved quantities for the following potentials $V = V(\mathrm{r})$ for a single particle in three-dimensional space.

(a) $V = V_0 = \text{const.}$

(b) $V = A_0/\sqrt{x^2 + y^2 + z^2}$

(c) $V = -b_0/2 \ln(x^2 + y^2)$

(d) $V = V(z)$

(e) $V = V(|\mathrm{r}|) - \vec{\sigma}_0 \cdot \mathrm{L}$ (L = angular momentum; $\vec{\sigma}_0$ is a fixed vector along z-axis)

(f) $V = -(q/c)\mathrm{v} \cdot \mathrm{A}(\mathrm{r})$, where $\mathrm{A} \equiv \frac{1}{2}\mathrm{B}_0 \times \mathrm{r}$, and B_0 is a fixed vector along the z-axis.

4 1) Consider a cylinder ($I = \frac{1}{2}MR^2$) beginning from rest and rolling down a plane inclined at an angle θ with respect to the horizontal. Find the position and velocity of the center of mass as a function of time by applying i) Newtonian and ii) Lagrangian mechanics. That is present two solutions to this (one) problem. Be sure to provide defintions, clear explanations and labeled sketches.

2) Too easy? Let's say there is this precocious senior Physics major who makes two very cogent observations about the above problem and thus objects to the "standard" solutions. He says

i) The expression

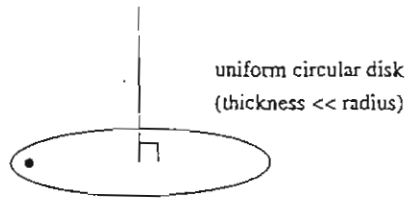
$$\sum \tau = I\alpha$$

is derived from Newton's second law and thus is applicable (only?) in inertial frames of reference. Surely a frame accelerating with the center of mass of the cylinder is not an inertial frame, so how can you write $\sum \tau = I\alpha$ about the axis of rotation as you solve the problem?

ii) A frictional (non-conservative) force acts on the rolling clinder (otherwise it would slip, not roll). It is therefore impossible to write down a Lagrangian for this system.

Please answer explicitly the objections of the student.

5. Two astronauts on the moon are playing "frisbee" with a *thin*, uniform disk. A small marking on the upper surface of the disk is present, as shown, and can be used to follow the rotation (spin) of the disk about its own axis. The astronauts notice that, when the disk is tossed slightly off-axis, a slight wobble (precession) results, and, in addition, the observed precession rate is directly proportional to the observed spin rate of the disk.



(a) Compute the *ratio* of precession rate to spin rate which is observed by the astronauts (appropriate to the case of a *slight* wobble).

(b) Sketch *accurately and to scale* the angular velocity ($\vec{\omega}$) and angular momentum (L) vectors in relation to the symmetry axis of the disk (again for the case of a *slight* wobble).

In your sketch, *resolve and label* the $\vec{\omega}$ vector into components which correspond to spin and precession rates, respectively.