Department of Physics

Preliminary Exam January 3–7, 2000

Day 1: Classical Mechanics

Monday, January 3, 2000

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your identification number (*not* your name!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permitted. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

Useful Information:

$$\begin{cases} \sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \\ \cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \\ \sin^2(x) + \cos^2(x) = 1 \end{cases}$$

A first-order differential equation of the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} + Py = Q,$$

where P and Q may be functions of x, has the solution:

$$y = \mathrm{e}^{-I} \int Q\mathrm{e}^{I}\mathrm{d}x + c\,\mathrm{e}^{-I},$$

where $I = \int P\mathrm{d}x$ and c is a constant of integration.

Problem 1

A mass M falls under gravity (force Mg) through a fluid whose viscosity is decreasing so that the retarding force is $F(v,t) = -2Mv(t)/(1\,\mathrm{sec} + t)$, where $v(t)$ is the speed of the mass M at time t.

(a) If the mass starts from rest, $v_0 = 0$, find (in terms of g) its speed, acceleration, and how far it has fallen at some later time t.

(b) Compare your results with those for the case of free-fall (where $F(v,t) = 0$) at a time $t = 1\,\mathrm{sec}$.

Problem 2

What is the frequency for small amplitude oscillations of a simple pendulum consisting of a mass M suspended by a massless rod of length $L = 2.0$ m

(a) in a laboratory ?

(b) in an elevator accelerating upward at a rate of $a = 2.0$ m/s^2 ?

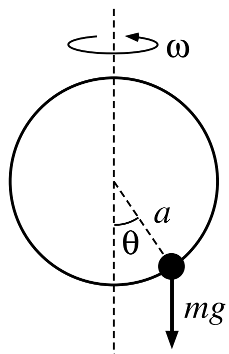
(c) in free-fall ?

Problem 3

As shown below, a bead of mass m slides without friction on a circular loop of radius a. The loop lies in a vertical plane and rotates about its vertical diameter with a constant angular velocity ω.

(a) The bead will undergo small oscillations about a stable equilibrium angular position θ_0, when the angular velocity ω becomes larger than a critical value ω_c. Write down Lagrange's equations of motion for the system and find ω_c and $\theta_0(\omega)$.

(b) Obtain the equations of motion for the small oscillations about θ_0 as a function of ω and find the period of the oscillations.



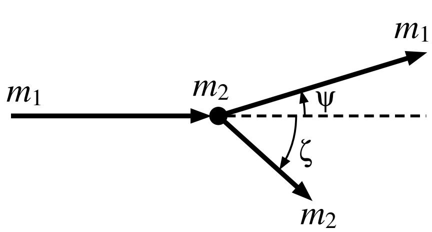
Problem 4

As shown below, particles of mass m_1 elastically scatter from particles of mass m_2 at rest.

(a) At what angle ψ should a particle detector be set to detect m_1 particles that lose 1/3 of their momentum?

(b) Over what range of m_1/m_2 is this possible?

(c) Calculate the scattering angle ψ for $m_1/m_2 = 1$.



Problem 5

A particle of mass m is moving in a plane under the influence of a potential, $V(r) = k/r^2$, where r is the distance from the origin.

(a) Write down the Lagrangian and the Hamiltonian of the system. [Hint: Use polar coordinates].

(b) Show that Lagrange's and Hamilton's equations of motions give the same set of equations of motion for the system.

(c) From Hamilton's equations, show that the angular momentum of the particle is conserved.

(d) Find the solutions of the equations for $k = -J^2/(2m)$, where J is the angular momentum.