Department of Physics

Preliminary Exam January 3–7, 2000

Day 2: Electricity, Magnetism, and Optics

Tuesday, January 4, 2000

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions: **Do problem 1 and choose four of the other five problems**.
2. Be sure to write your identification number (*not* your name!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permitted. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

Useful Information

For Spherical Coordinate System: $[\mathbf{r} \equiv (r, \theta, \phi)]$

$$\nabla\psi = \mathbf{e}_r \frac{\partial\psi}{\partial r} + \mathbf{e}_\theta \frac{1}{r}\frac{\partial\psi}{\partial\theta} + \mathbf{e}_\phi \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \mathbf{e}_r \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial\phi}\right] + \mathbf{e}_\theta\left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)\right] + \mathbf{e}_\phi\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

$$\left[\text{Note that } \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) \equiv \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)\right]$$

For Cylindrical Coordinate System: $[\mathbf{r} \equiv (\rho, \phi, z)]$

$$\nabla\psi = \mathbf{e}_\rho \frac{\partial\psi}{\partial\rho} + \mathbf{e}_\phi \frac{1}{\rho}\frac{\partial\psi}{\partial\phi} + \mathbf{e}_z \frac{\partial\psi}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \mathbf{e}_\rho\left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right] + \mathbf{e}_\phi\left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right] + \mathbf{e}_z\frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right]$$

$$\nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2}$$

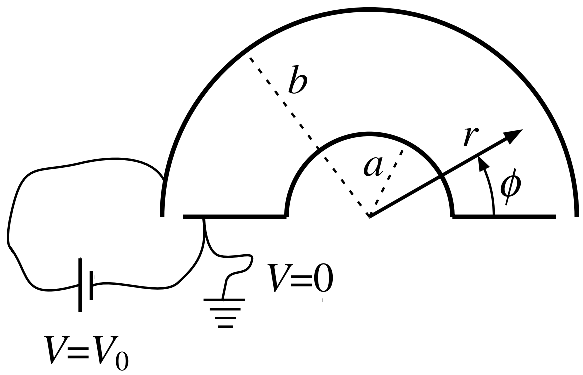
$$\int_{-1}^{1} P_l(x)P_{l'}(x)\mathrm{d}x = \frac{2}{2l+1}\,\delta_{ll'}$$

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = \tfrac{1}{2}(3x^2 - 1), \quad \cdots$$

Problem 1

A pair of electrodes has the geometry shown in the figure, extending infinitely far in the z direction (perpendicular to the plane of the paper). That is, in effect you can ignore the z direction. The two electrodes are connected across a potential difference so that

$$
\begin{aligned}
&V(r,0) = V(r,\pi) = 0, \quad a \le r < b \\
&V(a,\phi) = 0, \quad 0 \le \phi \le \pi \\
&V(b,\phi) = V_0, \quad 0 \le \phi \le \pi
\end{aligned}
$$



i) Find the potential $V(r,\phi)$ in the space bounded by the electrodes.

ii) Find the field $\mathbf{E}(r,\phi)$.

iii) How would the calculation be effected by having a uniform, homogeneous dielectric (such as water) fill the entire volume bounded by the electrodes?

iv) A (biological material) cell has a polarizability α. What is the force on the cell, as a function of r, placed along the line at $\phi = \frac{1}{2}\pi$?

Hint: Show that the force on a dipole in an electric field is: $\mathbf{F} = \mathbf{p}\cdot\boldsymbol{\nabla}\mathbf{E}$.

Note: *Do four of the following five problems. No extra credit will be given if solutions to all five problems are handed in; in that case, one of the solutions will be discarded.*

Problem 2

Each of two plates of a parallel plate capacitor has an area A and is separated from the other by a distance l. They are charged by placing a potential difference V across the plates. The plates are in a vacuum. Find the electrostatic force on one of the charged plates by the other.
Hint: Typically one can find the force, say, on the right plate by considering the change in the electrostatic potential energy stored in the field (between the plates) as the plate is moved a small distance δl and relating this change to the work done "against" the electrostatic force by an external agent. Just to keep the calculation honest consider such a calculation in two situations

i) with the battery disconnected from the circuit so that the charge on either plate is fixed;
ii) with the battery connected across the plates so that the potential difference between the plates is constant.

Note that in the latter case the battery is also an "external agent" which may do work on the charges.

Problem 3

Given the scalar and vector potentials

$$V(r, \theta, \phi, t) = 0, \qquad \mathbf{A}(r, \theta, \phi, t) = -\frac{qt}{4\pi\epsilon_0 r^2}\,\hat{\mathbf{r}},$$

where $\hat{\mathbf{r}}(\theta, \phi) \equiv \mathbf{e}_r$ is the unit radial vector.

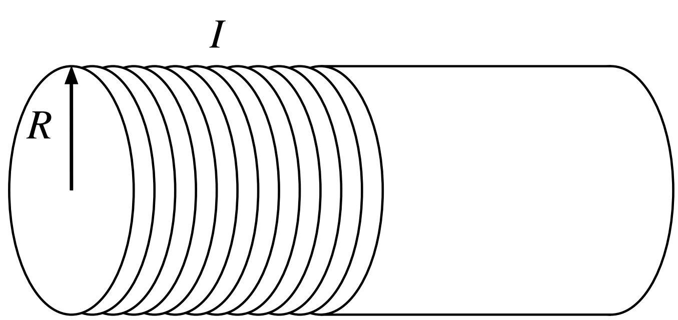
a) Find the electric and magnetic fields.
b) What kind of charge and/or current distributions(s) are associated with these potentials?
c) Are these potentials in the Coulomb gauge? Are they in the Lorentz gauge? Prove your answers.

Problem 4

Nuclear magnetic resonance (NMR) spectroscopy is a powerful technique for structural determination of molecules. To resolve the structure of large molecules (e.g. a protein with 1,500 atoms), a strong magnetic field is essential. In a 400 MHz NMR machine, the strength of the magnetic field is about 10 Tesla (T). Such a magnetic field is generated inside a long solenoid, consisting of n wound turns per unit length carrying a large current of 80 Ampères with superconducting wires (zero resistance).

(a) How many turns of wire per centimeter should be made in order to generate a 10 Tesla magnetic field with a current of 80 Ampères? ($\mu_0 = 4\pi 10^{-7}\,\mathrm{N\,A^{-2}}$, $1\,\mathrm{T} = 1\,\mathrm{N\,A^{-1}m^{-1}}$.)

(b) What is the torque on the magnetic dipole of the carbon isotope ^{13}C ($\mu_{^{13}\mathrm{C}} = 3.5\cdot 10^{-27}\,\mathrm{J\,T^{-1}}$)? Consider two situations: the dipole is parallel to the magnetic field, and the dipole is perpendicular to the magnetic field.



n turns per unit length, not all turns are shown.

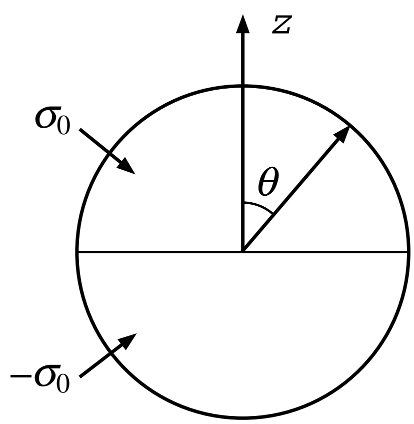
Problem 5

A spherical shell of radius R carries a uniform surface charge σ_0 on the upper hemisphere and $-\sigma_0$ on the lower hemisphere.

(a) What is the dipole moment of this charge distribution.

(b) Using the method of multipole expansion, find the electric potential of this charge distribution at a large distance (*keep the lowest non-zero term*).

(c) What is the electric field generated by this charge distribution (keep the dipole term only)?



Problem 6

Answer each of the following questions. Use diagrams when necessary to illustrate your answers.

(a) Describe three ways in which natural light can be polarized.

(b) Describe spherical and chromatic aberrations in thick lenses. In particular, explain why these aberrations appear and how they can be corrected.

(c) Explain why the sky is blue during midday and red at sunrise and sunset.