Department of Physics

Preliminary Exam January 3–7, 2000

Day 4: Thermodynamics and Statistical Physics

Friday, January 7, 2000

9:00 a.m. - 12:00 p.m.

Instructions:

- 1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions.
- 2. Be sure to write your identification number (not your name!) and the problem number on each problem sheet.
- 3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
- 4. If a question has more than one part, it may not be necessary to successfully complete one part in order to do the other parts.
- 5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
- 6. The use of electronic calculators is permitted. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

Problem 1

One mole of an ideal gas is expanded adiabatically from volume V_1 and pressure P_1 to volume V_2 and pressure P_2 . It is then compressed isobarically (at constant pressure) to volume V_1 . Finally the pressure is increased to P_1 keeping the volume constant.

- (a) Draw a P V diagram showing this cycle.
- (b) Show that the efficiency of the cycle is given by

$$\eta = 1 - \gamma \frac{V_2/V_1 - 1}{P_1/P_2 - 1}$$

where $\gamma = C_P/C_V$.

Problem 2

A 100 Ω resistor is held at a fixed temperature of 300 K. A current of 10 Ampères passes through the resistor for a total of 300 seconds.

- (a) What is the change in entropy of the resistor?
- (b) What is the change in entropy of the universe?
- (c) What is the change in internal energy of the Universe?
- (d) What is the change in the Helmholtz Free Energy of the universe?

Problem 3

Information: If a rubber band is stretched adiabatically, its temperature increases. For a rubber band, the work done on the rubber band as it stretches a distance dx under tension τ is given by τdx

- (a) If the rubber band is stretched isothermally, does its entropy increase, decrease, or stay the same?
- (b) If the rubber band is stretched adiabatically, does the internal energy increase, decrease, or stay the same?

Problem 4

A gas-filled tube is whirled about one end with angular velocity ω . Find the expression for the equilibrium density of the gas as a function of the distance r from the axis of rotation. You may take the gas to be an ideal gas of point particles.

Problem 5

A system of two energy levels E_0 and E_1 is populated by N particles at temperature T. The particles populate the energy levels according to the classical distribution law.

- (a) Derive an expression for the average energy per particle.
- (b) Compute the average energy per particle vs the temperature as $T \to 0$ and $T \to \infty$.
- (c) Derive an expression for the specific heat of the system of N particles.
- (d) Compute the specific heat in the limits $T \to 0$ and $T \to \infty$.

Problem 6

Beginning with the grand canonical ensemble, show that in a noninteracting boson gas at thermal equilibrium the average occupation number of a state with energy ϵ is given by

$$n(\epsilon, T) = \frac{1}{e^{(\epsilon - \mu)/kT} - 1}$$

where k is Boltzmann's constant and μ is the chemical potential of the gas.