Department of Physics

Preliminary Exam January 5–9, 2004

Day 1: Classical Mechanics

Monday, January 5, 2004

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permitted. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

Expressions needed:

$$g = 9.8\,\frac{\text{m}}{\text{s}^2}, \qquad \text{Density of water} = 1000\,\frac{\text{kg}}{\text{m}^3}.$$

Problem 1

A spherically shaped influenza virus particle, of mass 6×10^{-19} kg and with diameter 10^{-7} m, is in a water suspension in an ultra-centrifuge. It is 0.04 m from the vertical axis of rotation, and the angular speed of rotation is 10^3 revolutions per second. The density of the virus is 1.1 times that of water. [NOTE: In parts (b) and (c) below, account must be taken of the buoyancy effects. Think of the ordinary hydrostatics problem of a body completely immersed in a fluid of different density].

(a) Describe the total acceleration that the virus particle experiences in the rotating frame.

(b) Calculate the largest acceleration component on this particle.

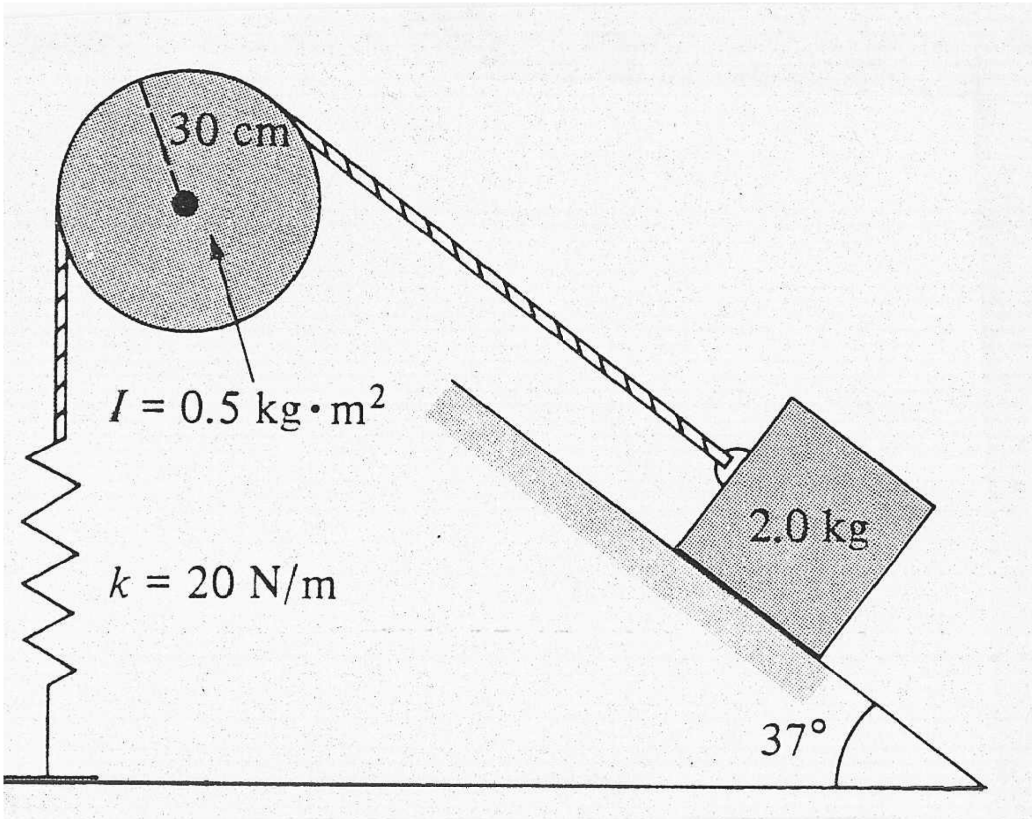
(c) Again from the standpoint of the rotating reference frame, what is the magnitude and direction of the net centrifugal force acting on the virus particle?

(d) The motion resulting from (b) is resisted by a viscous force given by $F_{\text{res}} = 3\pi\eta v d$, where d is the diameter of the particle and η is the viscosity of water, equal to $10^{-3}\,\frac{\text{kg}}{\text{m}\cdot\text{s}}$. What is the velocity v (magnitude and direction)?

(e) Determine the magnitude of the Coriolis force acting on the virus particle due to the velocity obtained in part (c).

Problem 2

The system shown in the figure below is released from rest with the spring in the unstretched position. The bottom of the spring is attached to the same surface on which the incline plane rests. If the moment of inertia of the pulley is $0.5\,\text{kg}\cdot\text{m}^2$ and assuming that friction can be neglected:



(a) How far will the mass slide down the incline?

(b) What will the speed of the mass be when the mass has slid 1.0 m down the incline?

(c) How far will the mass have slid when its speed is a maximum? What is this maximum speed?

Problem 3

Calculate the moment of inertial tensor for a thin sheet of uniform mass density in the shape of an equilateral triangle of side a about one of its corners. The sheet has a mass m and you may neglect its thickness. What are the principal moments? Identify the principal axes and illustrate them in a figure.

Problem 4: NON-CENTRAL FORCES AND ORBITS

Dark, microscopic dust particles, mostly rocky, fill the Solar System. Each particle orbits the Sun under its gravitational influence. However, for these dust particles, the pressure exerted by the Sun's light is also significant. The smallest particles can be blown out of the Solar System. In contrast, larger particles suffer orbital decay, and eventually fall into the Sun.

Photons flowing radially outward from the Sun strike such particles as they orbit at speeds $v \sim 10\,\mathrm{km/s}$. There are two important relativistic effects. First, in the rest frame of the particles, the photons arrive at an approximate angle $\theta = v/c$ slightly "ahead" of the particle-Sun vector (see Figure 1), where the speed of light is $c = 3 \times 10^8\,\mathrm{m/s}$. Second, a packet of photons having total energy ΔE will change the linear momentum of a mass m by an amount $\Delta p = \Delta E/c$ if fully absorbed.

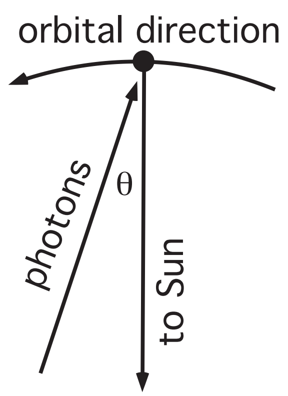


Fig. 1

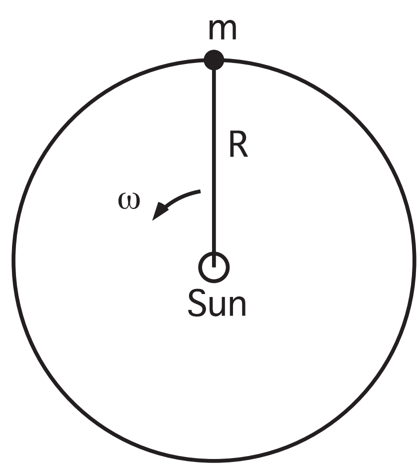


Fig. 2

(a) Refer to Figure 2. What is the orbital angular momentum L of a dust particle, of mass m and radius r, following a circular orbit of radius R and angular velocity ω around the Sun?

(b) Derive an algebraic expression for the change ΔL in the dust particle's orbital angular momentum due to the absorption of a photon packet of energy ΔE. Express your answer in terms of four of the foregoing quantities.

(c) Now express ΔL in terms of L and other quantities.

(d) For simplicity, assume that ΔE, the amount of energy striking the dust particle's sunny side each second, is constant. Then use the result from (c) to derive an approximate algebraic expression for amount of time the particle will take to fall into the Sun.

(e) Evaluate that expression for a typical dust particle of radius $10^{-6}\,\mathrm{m}$ and density $3000\,\mathrm{kg\,m^{-3}}$ initially orbiting at the Earth's distance from the Sun. The flux of radiant energy from the Sun at that distance is $F = 1400\,\mathrm{J\,m^{-2}\,s^{-1}}$. Express the answer in years ($1\,\mathrm{yr} = 3.2 \times 10^{7}\,\mathrm{s}$).

Problem 5

Two point masses, m_1 and m_2, are connected by a string that acts as a Hooke's law spring of force constant k. One particle (m_1) is free to move without friction on a smooth horizontal plane surface, while the other (m_2) hangs vertically down from the string through a hole in the surface. The unstretched length of the string is l.

(a) Set up the Lagrangian for the system and write down the equations of motion. Identify the constants of motion in the problem.

(b) Find the condition for steady motion in which m_1 on the plane rotates uniformly at constant distance from the hole.

(c) Investigate the small oscillations in the radial distance of m_1 from the hole and obtain the oscillation frequency.

(d) Construct the Hamiltonian for the system in terms of the coordinates and the canonical momenta.