Department of Physics

Preliminary Exam January 5–9, 2004

Day 2: Electricity, Magnetism, and Optics

Tuesday, January 6, 2004

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permitted. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

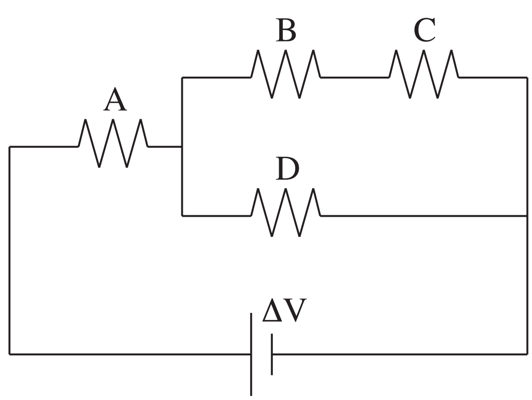
Useful Information:

$$k = 1.38 \times 10^{-23}\,\text{J/K}$$

$$h = 6.63 \times 10^{-34}\,\text{J}\cdot\text{s}$$

Problem 1

In the figure below, light bulbs are denoted by resistor symbols. Assume that bulbs A-D have equal resistances. When bulb C is removed from its socket, what happens to brightness of bulb D? Formulate your answer by calculating the power dissipated by bulb D both before and after the removal of bulb C.



Problem 2

Two conducting bodies are brought into contact when the first is uncharged and the second has charge Q. They are then separated, leaving charge q on the first body. The second body is then recharged to Q and they are brought into contact again. This process is repeated indefinitely. What is the final charge on the first body?

Problem 3

It is known that a 1-mW He-Ne laser is hundreds of times "brighter" than the sun. To develop an appreciation for the enormous difference between the radiance of lasers and typical thermal sources consider the following problem.

a) Small gas lasers typically have power outputs of 1 mW. By contrast, neodymium-glass lasers, such as those designed for the production of laser-induced fusion, boast of power outputs near 10^{14} W. Using these two extremes and an *average energy of* 10^{-19} *J per visible photon*, determine the approximate range of photon output from the lasers.

b) For comparison, consider a broadband thermal source with a radiating surface equal to that of the beam waist of a 1-mW He-Ne laser with diameter of 0.5 mm (or area of $A = 2 \times 10^{-7}\,\mathrm{m}^2$). Let the surface emit radiation at a wavelength of 633 nm, with a line width $\Delta\lambda = 100\,\mathrm{nm}$ and temperature $T = 1000\,\mathrm{K}$. Determine the photon output rate of this thermal source.*

c) Compare the magnitude of the thermal rate calculated in part (b) to the photon output per second for both the lower- and higher-power lasers of part (a).

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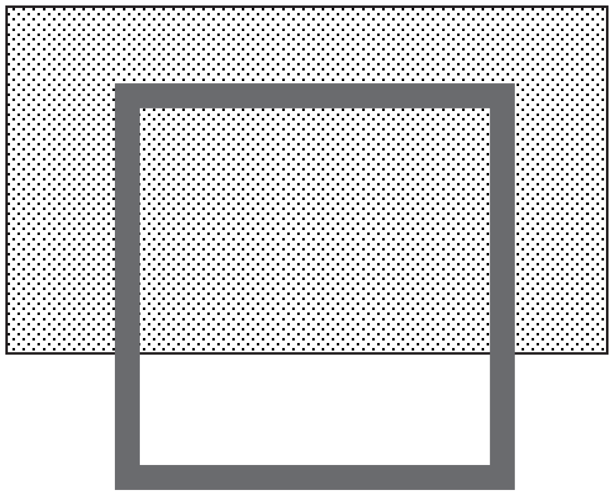
* *Hint:* One can obtain the expression for thermal photons per second from the spectral energy density of blackbody radiation given as

$$P(\nu) = \frac{2\pi h\nu^3}{c^2}(e^{h\nu/kT} - 1)^{-1}$$

in units of power per unit area per unit frequency range by doing a unit analysis with the information provided in this problem.

Problem 4

A square loop is cut out of a sheet of copper. It is then placed so that the top portion is in a uniform magnetic field $\mathbf{B}$, and allowed to fall under gravity. See the figure, in which $\mathbf{B}$ points into the page.



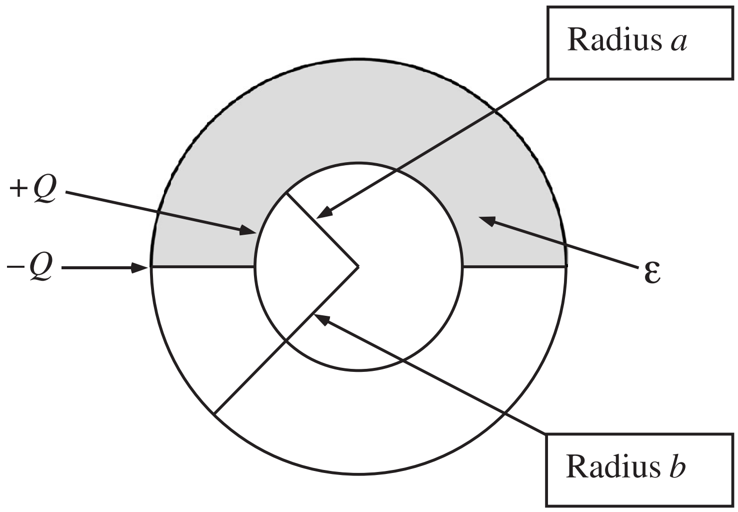
In answering the following two questions you will want to make, and briefly explain, some reasonable approximations.

(a) Find the velocity of the loop as a function of time.

(b) Take the resistivity of Cu to be $2 \times 10^{-8}\,\Omega\mathrm{m}$ and the density of Cu to be $1 \times 10^{4}\,\mathrm{kg/m^3}$. If the magnetic field is $1\,T = 1\,\mathrm{N/(Am)}$, find the terminal velocity of the loop.

Problem 5

Two concentric conducting spheres of inner and outer radii a and b, respectively, carry charges of $\pm Q$. The empty space between the spheres is half-filled by a hemispherical shell of dielectric material (of dielectric constant ε), as shown in the figure.



(a) Find the electric field everywhere between the spheres.

(b) Calculate the surface-charge distribution on the inner sphere.

(c) Calculate the polarization-charge density induced on the curved surface of the dielectric at $r = a$.

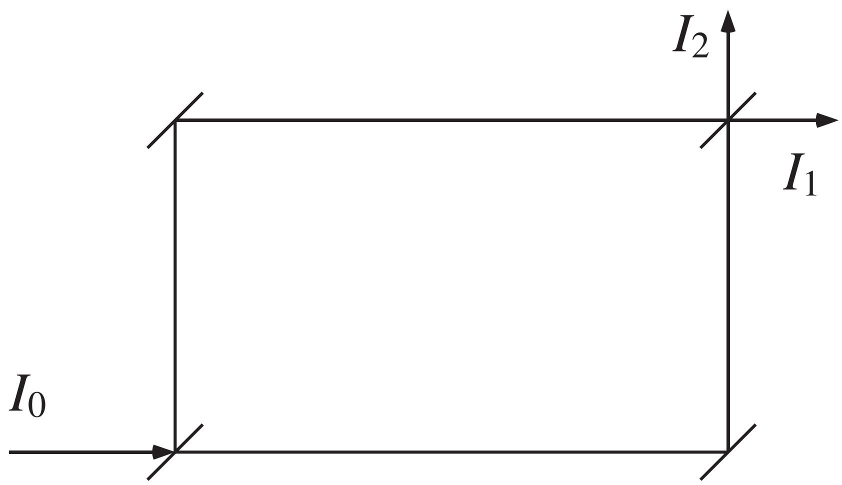
Note: The general solution of the Laplace equation for potential for a boundary-value problem in spherical coordinates can be written in terms of spherical harmonics and powers of r. The potential is

$$\Phi(r,\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left[A_{lm}r^{l} + B_{lm}r^{-(l+1)}\right]Y_{lm}(\theta,\phi);$$

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{(2l+1)\,(l-m)!}{4\pi(l+m)!}}\,P_l^m(\cos\theta)e^{im\phi}.$$

Problem 6

The interferometer sketched below is made up of two perfect reflectors and two perfect 50/50 beamsplitters, so that the beams (treatable as plane waves) of red light following the two paths have equal amplitudes inside the interferometer. It is carefully set up so that the two paths are of equal length. As a result, the two transmitted beams have relative intensities of $I_1/I_0 = 100\%$ (the two combined beams add in phase) and $I_2/I_0 = 0$ (the two combined beams add out of phase).



Now a slab of preliminia is inserted into one of the two paths. Preliminia is a special type of glass with the remarkable property that at the *exact* wavelength $\lambda = 2\pi \times 10^{-7}$ m (about 628 nm) its index of refraction is *exactly* $n_\mathrm{r} = 1 + (2\pi/10)$, or about 1.628. The index of refraction is complex; that is, $n = n_\mathrm{r} + in_\mathrm{i}$, where $n_\mathrm{i} = 5.00 \times 10^{-7}$. If the slab of preliminia has a thickness of *exactly* 10 cm (the reflections from its faces are made negligible by antireflection coatings), find I_1/I_0 and I_2/I_0.