

Department of Physics
Preliminary Exam January 3–6, 2006
Day 2: Electricity, Magnetism and Optics
Wednesday, January 4, 2006
9:00 a.m.–12:00 p.m.

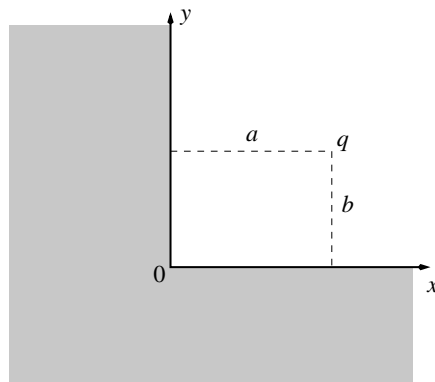
Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

Note—All questions to be attempted. All questions are of equal weight.

Problem 1

A point charge q is located in the x - y plane at a distance b from the x -axis and a from the y -axis near two grounded conducting planes intersecting at right angles as shown in the figure below. The z -axis lies along the line of intersection of the planes. Using the method of images:



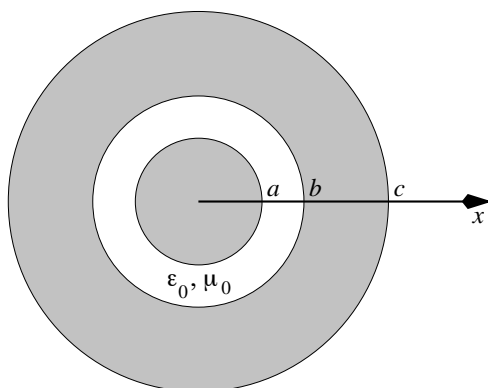
- Find and justify the image charges that, together with q , give the potential at all points in the vacuum region, $x \geq 0$, $y \geq 0$, and $-\infty \leq z \leq \infty$.
- Find the potential $\phi(x, y, z)$ in the vacuum region.
- Find $E_y(x, y, z)$.
- Verify that E_y vanishes on the conducting plane for which it is a tangential component.
- Find the surface charge density σ_f induced on the plane for which E_y is an appropriate component to use.
- What is the sign of σ_f ? Explain.

Problem 2

Derive the force on a dipole in an electric field. What is the force if the electric field is uniform? Calculate the force on a dipole with dipole moment \mathbf{p}_1 and located at \mathbf{r}_1 due to the field produced by a second dipole \mathbf{p}_2 located at \mathbf{r}_2 . What is the force if \mathbf{p}_1 and \mathbf{p}_2 are perpendicular to the line joining \mathbf{r}_1 and \mathbf{r}_2 ?

Problem 3

The figure below shows the cross section of a long coaxial cable. The center conductor ($r \leq a$) carries a current I_0 in the direction out of the page. The outer conductor ($b \leq r \leq c$) carries the return current I_0 into the page. The currents are uniformly distributed in the conductors. Between the conductors there is an insulating material with permittivity ϵ_0 and permeability μ_0 . Find \mathbf{B} for all r and sketch a plot of \mathbf{B} versus r .



Problem 4

Derive an expression for the intensity profile, on a distant screen, for multiple-slit interference (diffraction grating). Assume normal incidence of light with wavelength λ , on slits of negligible width having a spacing d , and small angles of deviation. Use your result to show that the wavelength resolving power of a grating spectrometer is proportional to the product of the order m and the number N of slits illuminated:

$$\frac{\lambda}{\Delta\lambda} \propto mN.$$

Problem 5

Consider two electromagnetic waves given by

$$\mathbf{E}_1 = E_0 \vec{\epsilon}_1 e^{ikz - i\omega t} + \text{c.c.} \quad \mathbf{E}_2 = E_0 \vec{\epsilon}_2 e^{ikz - i\omega t + i\phi} + \text{c.c.}$$

Calculate the interference of these two plane waves. Find the condition on polarization vectors for no interference. Find the visibility of the interference. Note that the vectors $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ can be complex. In all the calculations drop terms like $e^{\pm 2i\omega t}$.

Problem 6

Consider a uniform magnetic field \mathbf{B}_0 along the z -axis. Give any two choices of the vector potential \mathbf{A} that satisfy the basic relation between \mathbf{B} and \mathbf{A} . Show that the two choices differ by the gradient of a scalar function f . Find this function f for your choices.