Department of Physics

Preliminary Exam January 3–6, 2006

Day 3: Quantum Mechanics and Modern Physics

Thursday, January 5, 2006

9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

Attempt all five problems. Each problem carries 20 points.

Problem 1

In 1913 Bohr proposed that the angular momentum of an electron orbiting around the nucleus of a Hydrogen atom was quantized in units of $\hbar$.

(a) Determine expressions for the energy of the ground state and the first excited state of Hydrogen based on this model.

(b) In the case of Hydrogen, what are the principal differences between the Bohr model and the solution obtained by solving the Schrödinger equation?

(c) One of Bohr's original motivations for this work was to understand why the orbit of an electron did not collapse into the nucleus of an atom (an accelerating charge loses energy via the emission of electromagnetic radiation). Provide another explanation for the stability of the electron's dynamics.

Problem 2

An astronaut in a spaceship of length L (as measured by the astronaut) sets up a coordinate system S' with its origin at the center of the ship and its x'-axis parallel to the length of the ship. At time $t = t' = 0$, the astronaut switches on a light bulb located at the center of the ship.

(a) At what time(s) in S' will the light arrive at the front and back of the ship?

(b) Consider a second reference frame S (origin coincident with S' at $t = t' = 0$, x and x' axes parallel) in which the ship is observed to travel at velocity $u = \frac{3}{5}c$ in the x-direction. At what time(s) will the light arrive at the front and back of the ship in S?

(c) What is the length of the ship in reference frame S?

(d) What is the space-time interval between the events corresponding to the arrival of the light at the ends of the ship (i) in S; (ii) in S'?

(e) What is the *proper length* of the ship? What is the *proper time* between the arrival of the light at either end of the ship? Explain your answers.

Problem 3

Consider the following matrix representations for the angular momentum operators:

$$\hat{L}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \hat{L}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \hat{L}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) What are the possible values one can obtain if L_z is measured?

(b) Take the state in which $L_z = \hbar$. In this state what are $\langle \hat{L}_x \rangle$ and $\langle \hat{L}_x^2 \rangle$?

(c) Find the normalized eigenstates and the eigenvalues of $\hat{L}_x$ in the $\hat{L}_z$ basis.

(d) If the particle is in the state with $L_z = -\hbar$ and L_x is measured, what are the possible outcomes and their respective probabilities?

Problem 4

Consider a particle of mass m in a one-dimensional infinite square well of width a. The eigenvalues of the Hamiltonian are $E_n = \dfrac{n^2\pi^2\hbar^2}{2ma^2}$ and the eigenvectors are $|\varphi_n\rangle$.
At $t = 0$ the particle is in the state

$$|\Psi(0)\rangle = a_1|\varphi_1\rangle + a_2|\varphi_2\rangle + a_3|\varphi_3\rangle + a_4|\varphi_4\rangle\,.$$

(a) When the energy of the particle in the state $|\Psi(0)\rangle$ is measured, what is the probability of finding a value smaller than $\dfrac{3\pi^2\hbar^2}{ma^2}$?

(b) What are the mean (expectation) value and the root-mean square deviation (uncertainty) of the energy of the particle in the state $|\Psi(0)\rangle$?

(c) What is the state vector $|\Psi(t)\rangle$? Do the results obtained in (a) and (b) at $t = 0$ remain the same at an arbitrary time t?

(d) When the energy is measured, the value $\dfrac{8\pi^2\hbar^2}{ma^2}$ is obtained. What is the state of the system after the measurement? What is the result if the energy is measured again?

Problem 5

Consider a particle of mass m moving in a one-dimensional potential well

$$V(x) = -g\,\delta(x)$$

where g is a positive constant.

(a) Integrate the time-independent Schrödinger equation to show that the first derivative of the eigenfunction $\varphi(x)$ is discontinuous at $x = 0$.

(b) Calculate the energy eigenvalues of the possible bound states and the corresponding normalized eigenfunctions.