Department of Physics

Preliminary Exam January 3–6, 2006

Day 4: Thermodynamics and Statistical Physics

Friday, January 6, 2006

9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

Physical Constants:
$h = 6.626 \times 10^{-34}\,\mathrm{J\,s} = 4.136 \times 10^{-15}\,\mathrm{eV\,s}$
$m_e = 9.109 \times 10^{-31}\,\mathrm{kg}$

Problem 1

A tank of volume $0.5\,\mathrm{m}^3$ contains O_2 at an absolute pressure of $1.5 \times 10^6\,\mathrm{Pa}$ and a temperature of $20\,^\circ\mathrm{C}$. Assume that oxygen behaves like an ideal gas.

(a) How many kilomoles of oxygen are there in the tank?

(b) How many kilograms?

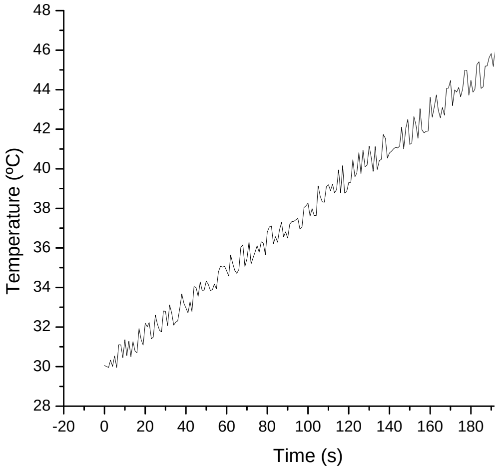
(c) Find the pressure if the temperature is increased to $500\,^\circ\mathrm{C}$.

(d) At a temperature of $20\,^\circ\mathrm{C}$, how many kilomoles can be withdrawn from the tank before the pressure falls to 10 percent of the original pressure?

(e) What is the physical origin of the pressure in the system?

Problem 2

An experiment is performed with 2 moles of gas in an adiabatic container of $0.5\,\mathrm{m}^3$ provided with a piston, initially locked in place. The gas is initially at $30\,^\circ\mathrm{C}$ and at a pressure of $5.04\,\mathrm{kPa}$. A heater is introduced in the container and the current is switched on at the instant $t = 0\,\mathrm{s}$, dissipating a power of $1\,\mathrm{W}$. A thermocouple registers the following temperature profile as a function of the time after time zero:



(a) Based on the data presented in the figure above, obtain an estimate for the heat capacity *and* for the specific heat capacity at constant volume for this gas.

(b) If the experiment is repeated under the same initial conditions, but with the piston unlocked, i.e., able to move in such a way that the pressure is constant, would the temperature be higher or lower than in the previous case at $t = 200\,\mathrm{s}$? Explain.

(c) Calculate the final temperature of the gas in the situation explained in part (b) above. (Assume an ideal gas.)

Problem 3

In the following situations, calculate the variation in entropy *in the system* and *in the universe.*

(a) A system consisting of 1 kg of water at 20 °C is brought in contact with a heat reservoir at 50 °C. The temperature of the system increases until it reaches the temperature of the reservoir. Consider the specific heat capacity as a constant equal to $1\,\mathrm{cal\,g^{-1}\,K^{-1}}$ ($4.18 \times 10^3\,\mathrm{J\,kg^{-1}\,K^{-1}}$ in SI units).

(b) Free expansion: an isolated system consisting of 1 kmol of an ideal gas is restricted to occupy half of the volume of an insulating container due to an insulating partition. The partition is quickly removed and the gas is allowed to expand until it occupies the whole volume and equilibrium is reestablished.

(c) A reversible engine operating between the temperatures of 400 K and 300 K performs n cycles.

Problem 4

A mass (weight $W = mg$) is suspended from the ceiling by a rubber band. (Take the $+z$-direction to be vertically *downward* such that z measures vertical displacement of the mass below the ceiling.) Model the rubber band as a chain of N line segments, each of negligible mass and of length L_{seg}; each segment can be randomly oriented either *upward* or *downward* (with internal energy of the rubber band being independent of the relative orientation of adjacent segments). Hence, the end-to-end length of the rubber band is

$$z = L_{\text{seg}} \sum_{i=1}^{N} \sigma_i$$

where $L_{\max} = N L_{\text{seg}}$ and $\sigma_i = \pm 1$.

(a) Draw and label a figure or set of figures depicting this system.

(b) Write down an expression for the potential energy of this system.

(c) Calculate the mean length of the rubber band in thermodynamic equilibrium as a function of W.

(d) Sketch this dependence for three different temperatures. What is the physical origin of the elastic behavior of the rubber band in this model?

Problem 5

Conduction electrons in metals can be approximately treated as a quantum ideal gas. $D(E)$ denotes the density of states of conducting electrons in a metal, and ε_F the Fermi energy. At the Fermi energy, $D(\varepsilon_F) \neq 0$.

(a) By noting that the energy quantization of an ideal gas in a rectangular box is

$$E(n_x, n_y, n_z) = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right),$$

derive the following expression for $D(E)$ for a 3-dimensional Fermi gas as a function of energy, volume of the metal, and mass of the electron:

$$D(E) = \frac{\pi V}{2}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2}.$$

(b) Give an expression for the total number of electrons in the system, N, at temperature $T = 0$ in terms of ε_F and $D(E)$.

(c) For the case of sodium metal there are approximately 2.5×10^{22} conduction electrons/cm^3. Calculate an approximate value of the Fermi energy in sodium metal in eV.