

**Department of Physics**  
**Preliminary Exam January 2–6, 2007**  
**Day 1: Classical Mechanics**  
**Monday, January 2, 2007**  
**9:00 a.m. – 12:00 p.m.**

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**Instructions:**

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

### Problem 1 (The Lagrangian & Angular Momentum Conservation)

A star of mass  $M$  is orbited by a planet of mass  $m \ll M$  that follows an elliptical path. The planet's motion is governed solely by the star's gravity.

Draw a picture of the system showing polar coordinates  $(r, \theta)$  centered on the star, and then derive expressions for the planet's

- (a) kinetic energy  $T$  and potential energy  $V$ .
- (b) Lagrangian  $L$ .
- (c) equations of motion in  $r$  and  $\theta$  (use the Lagrangian).
- (d) areal velocity  $dA/dt$ , the rate at which the rotating radius vector pointing from the star to the planet sweeps out area element  $dA$  during a short time  $dt$ . Its units are area/time. Base this derivation on a suitably labeled diagram.

Finally,

- (e) use your results from (c) and (d) to demonstrate that the planet's areal velocity is constant. This is Kepler's second law of planetary motion, discovered in 1619.

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### Problem 2

A simple pendulum consists of a bob of mass  $m$  suspended by an inextensible (and massless) string of length  $l$ . From the bob of this pendulum is suspended a second, identical pendulum. Consider the case of small oscillations, calculate the characteristic frequencies.

### Problem 3 (Conservation Laws and Collisional Cross Section)

Earth continuously collides with small bodies such as meteoroids, comets, and asteroids as it orbits the Sun. The Earth's gravity causes the cross section for such collisions to be larger than Earth's frontal area,  $\pi R^2$ . In particular, this cross section depends on the relative velocity of the Earth and small body (at infinity).

The diagram below depicts the approach of a small body of mass  $m$  at a very large distance from Earth with impact parameter  $b$  and relative velocity  $v_\infty$ . The Earth has mass  $M$  and radius  $R$ . The approaching body is negligibly small in size compared to  $R$ .

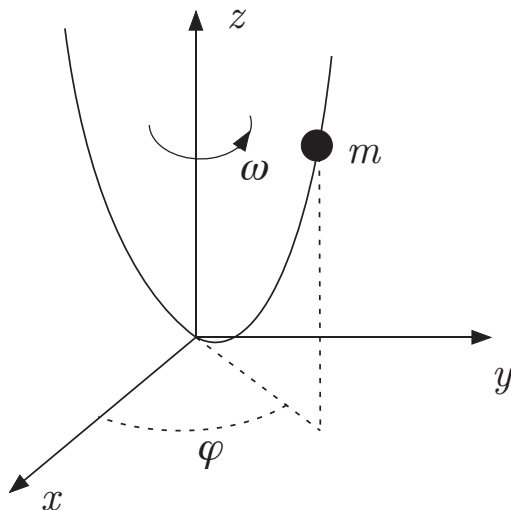
Applying the laws of momentum and energy conservation, compute the velocity-dependent impact parameter  $b$  in terms of the given parameters, where  $b$  is the largest impact parameter that will result in a (grazing) collision with Earth at impact speed  $v_f$ .

- (a) First, draw a picture of the geometry of such a grazing collision, including the vectors  $\mathbf{v}_\infty$  and  $\mathbf{v}_f$ .
- (b) Apply conservation of angular momentum and of energy to derive a velocity-dependent expression for  $b$  in terms of  $v_\infty$  and other given quantities. Clearly indicate where you are applying each conservation law.
- (c) Numerically evaluate the ratio  $(b/R)^2$  of the gravitational cross section to the Earth's frontal area, given that  $M = 5.97 \times 10^{24} \text{ kg}$ ,  $R = 6,380 \text{ km}$ ,  $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$ , and  $v_\infty = 30 \text{ km/s}$  (the Earth's average orbital velocity relative to the Sun, hence a representative value).



### Problem 4

Find Hamilton's equations of motion of a bead of mass  $m$  sliding on a frictionless wire under the influence of gravity. The wire has parabolic shape and rotates with constant angular velocity  $\omega$  about the  $z$  axis.



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### Problem 5

Consider a particle of mass  $m$  moving in a potential  $V(\mathbf{r}) = -\frac{km}{|\mathbf{r}|^n}$  with  $n$  arbitrary.

- Show that the angular momentum  $\mathbf{L}$  of the particle is a constant of motion.
- Show that the motion of the particle is confined to a plane.
- Find the effective potential for an equivalent one-dimensional problem.
- For orbital motion, find the range  $n$  for which the particle will be stable with respect to small perturbative oscillations in the radial direction.