Department of Physics

Preliminary Exam January 2–6, 2007

Day 2: Electromagnetism and Optics

Wednesday, January 3, 2007

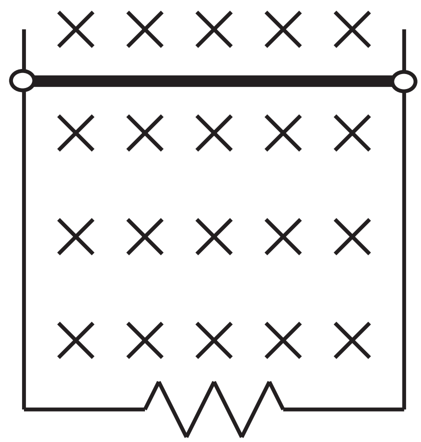
9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

Problem 1

Find the terminal velocity of the falling, horizontal bar. This bar slides freely on vertical wires that are connected at their base by a third wire of resistance R. The bar has length l and mass m. A uniform magnetic field of magnitude B is directed perpendicular into the page.



Problem 2

(a) An electron is in a vacuum and sits a distance d from a solid, infinite plane of silver. Determine the electrostatic force on this particle.

(b) A Cl^- ion immersed in water ($\varepsilon_{H_2O} = 80$) is a distance d from a flat interface (of negligible thickness) separating an adjacent volume of silicone oil ($\varepsilon_{oil} = 2.5$). Determine the electrostatic force on this ion.

Problem 3

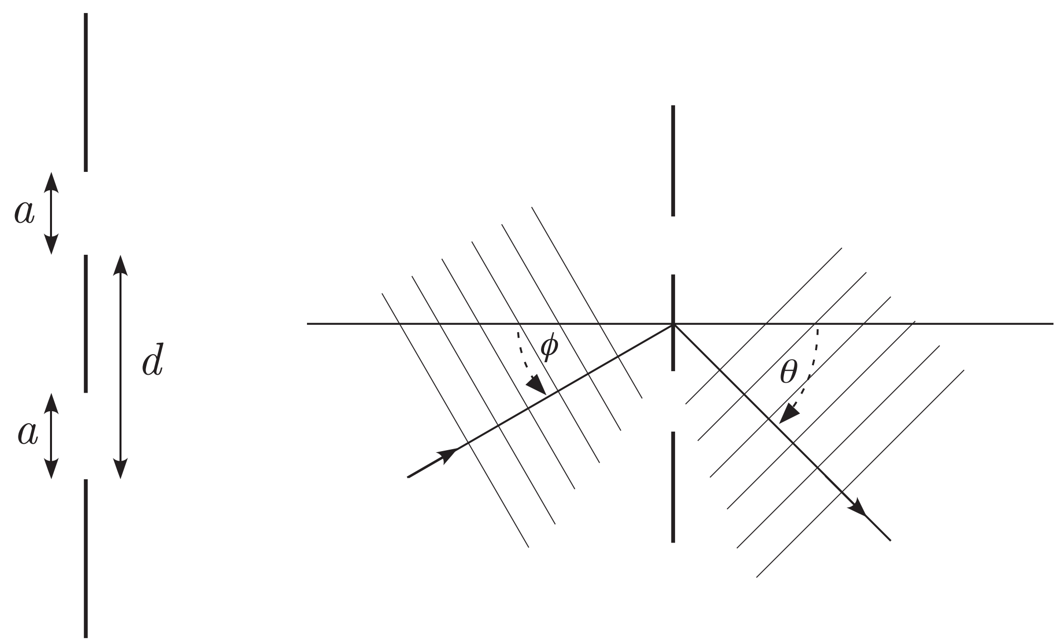
Design the Mirage, the optical illusion, where a penny placed on the surface of a concave mirror produces an image of the penny at the position of a hole in an otherwise identical concave mirror, inverted, and placed on top. Given the separation of the object and its image is 10 cm, what should the curvature of the mirrors be?

Problem 4

Consider a plane wave of wavelength λ incident at an angle ϕ with respect to the normal of a double slit. The slit opening is width "a" and the repeat distance is "d". (See the two figures.)

(a) Show that the intensity I as a function of angle θ in a plane at infinity (or in the focal plane of a converging lens, which is not shown in the figures) is

$$I \propto 4E_0^2 \frac{\sin^2[(\pi a/\lambda)(\sin\phi + \sin\theta)]}{\{(\pi a/\lambda)(\sin\phi + \sin\theta)\}^2} \cos^2[(\pi d/\lambda)(\sin\phi + \sin\theta)].$$



(b) Consider the case of normal incidence, $\phi = 0$. Let's say $(d/\lambda) = 10$ and $(a/d) = \frac{1}{4}$. The "interference" maxima may be labeled by an index m, $m = 0, \pm 1, \pm 2, \ldots$, with the central maximum I_0. Find the relative intensities (I_m/I_0) for $m = 2, 3, 4, 5, 6$ and discuss the result.

Problem 5

A (very) long hollow metal cylinder (a can) is divided in two by small gaps at $\phi = \pm\frac{1}{2}\pi$ with the right half at $+\frac{1}{2}V_0$ and the left half at $-\frac{1}{2}V_0$.

Find the electrical field at points in the yz plane ($\phi = \pm\frac{1}{2}\pi$) "across the gap" and the surface charge density on the inside surface of each of the two halves of the cylinder. (Note: supply the logic and answer the questions below).

(a) Consider Laplace's equation in cylindrical coordinates

$$\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\right\} V(r,\phi,z) = 0$$

and show the appropriate general solution (independent of z) to be

$$\begin{aligned} V(r,\phi) &= \alpha_0 + \beta_0\phi + \gamma_0 \ln(r) + \delta_0\phi\ln(r) \\ &\quad + \sum_{m=1}^{\infty}\{\alpha_m r^m \cos(m\phi) + \beta_m r^m \sin(m\phi) \\ &\qquad + \gamma_m r^{-m}\cos(m\phi) + \delta_m r^{-m}\sin(m\phi)\}. \end{aligned}$$

(b) Argue that for the interior of the can ($0 \le r \le a$) the expansion must be written

$$V(r,\phi) = \alpha_0 + \sum_{m=1}^{\infty}\{\alpha_m r^m\cos(m\phi) + \beta_m r^m \sin(m\phi)\}$$

(c) and in our particular problem the expansion further reduces to

$$V(r,\phi) = \alpha_0 + \sum_{m=1}^{\infty} \alpha_m r^m \cos(m\phi).$$

(d) If you have time show that $\alpha_m = 0$ for m is even and for odd m:

$$\alpha_m = \frac{2V_0}{m\pi a^m}(-1)^{\frac{1}{2}(m-1)}.$$

(e) Find $\mathbf{E}(r)$ in the yz plane (at $\phi = \frac{1}{2}\pi$ and $\phi = -\frac{1}{2}\pi$) and σ_{true} on the surfaces of the divided can.

Note the sums can be written in closed form, but you are not asked to do so. Of course, if you have nothing better to do, ...

Problem 6

Two positively charged particles, each having a charge Q, are each moving with a speed v with respect to the lab frame and at right angles to one another, as shown in the figure below. Indicate on the figure below the electrical and magnetic action-reaction forces that each particle exerts on the other. Comment on your result as it pertains to Newton's third law and discuss the assumptions which are the basis of your result. Comment further on what modifications (if any) need be introduced to get the physics correct.