Department of Physics

Preliminary Exam January 2–6, 2007

Day 3: Quantum Mechanics and Modern Physics

Friday, January 5, 2007

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

Physical Constants for Exam:

$$c = 2.998 \times 10^8 \text{m/s}$$
$$e = 1.602 \times 10^{-19} \text{C}$$
$$m_{\text{e}} = 9.109 \times 10^{-31} \text{kg}$$

Problem 1

The following problem involves a 1-d harmonic oscillator with spring constant k and mass m. Such a system with potential $V(x) = \frac{1}{2}kx^2$ can provide a good model for many physical systems.

(a) One of the wavefunctions of the 1-d harmonic oscillator is given by

$$\psi(x) = A\frac{x}{b}\exp\left(-\frac{x^2}{2b^2}\right),$$

where A and b are constants. Verify that this is indeed a solution of the 1-d Schrödinger equation, and determine b and the energy of the corresponding state in terms of k and m. *Explain* how you would go about finding A.

(b) Draw a rough sketch of the wavefunction with the 5^{th} lowest energy state in relation to the potential.

(c) Draw a rough sketch of the quantum probability distribution for a state with very large energy. Compare this distribution to the classical probability distribution of a particle undergoing simple harmonic motion.

Problem 2

A graduate student is assigned the task of carrying out an experiment on the physics of objects moving close to the speed of light. In this experiment a metal wire of proper length 10 cm is accelerated to a constant speed of $0.8\,c$ and then put through a pipe of length 7 cm. At either end of the pipe are a laser and a photo detector. If a pulse of laser light can pass unobstructed across the diameter of the pipe then the detector will register a "click"; if the light beam is blocked by the wire there will be no "click".

(a) Calculate the contracted length of the wire as seen by the student. Hence explain how if both lasers emit a short pulse of light just as the student sees the rear of the wire disappear into the pipe, it is possible for both detectors to register a click.

(b) Now view this scenario from the perspective of an observer traveling with the wire. To this observer the pipe appears even shorter than 7 cm, making it seemingly impossible for the 10 cm long wire to fit within the pipe so that the light pulses can be detected. Use the Lorentz transformations to explain how it is possible to resolve the apparent paradox.

Problem 3

Recall the expression for time evolution of an expectation value,

$$\frac{\mathrm{d}\langle A\rangle}{\mathrm{d}t} = \frac{\mathrm{i}}{\hbar}\langle[\hat{H}, \hat{A}]\rangle.$$

Consider the one-dimensional harmonic oscillator, whose Hamiltonian is given by

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2\hat{x}^2}{2}.$$

Show that the expectation value of x obeys the classical harmonic oscillator equation of motion,

$$\frac{\mathrm{d}^2\langle x\rangle}{\mathrm{d}t^2} = -\omega_0^2\langle x\rangle.$$

Problem 4

Photoelectrons are emitted when light is shone on a clean gold surface in vacuo. The table below gives experimental values for the wavelength, λ, of the light, and of the potential difference, V_s, which must be applied between gold and the collecting plate to suppress the photocurrent for incident light at the wavelength indicated. Show all work for each part below.

(a) Write down the relationship between wavelength, λ, the potential difference, V_s, and the work function ϕ_0 of gold.

(b) Assuming standard values for the other constants involved, use the data below to calculate an estimate for Planck's constant, h, in SI (MKSA) units.

(c) Also, use the same data to estimate the work function ϕ_0 of gold.

λ (nm)	216	184	160	142
V_s (V)	1.00	2.00	3.00	4.00

Problem 5

An electron of mass m is in a one-dimensional infinite square well of width a. The potential energy can be expressed as:

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ 0, & 0 < x < a \\ \infty, & x \geq a \end{cases} .$$

Using the one-dimensional Schrödinger equation (show all work):

(a) Derive an expression for the energy of an electron in its n^{th} lowest state, with $n = 1$ corresponding to the lowest energy eigenvalue.

(b) Determine where the electron wavefunction has the highest probability of being found for $n = 1$ and $n = 2$.

(c) Find the probability $P(\xi)$ that an electron in the n^{th} lowest state is found in the interval $0 < x < \xi$.

Problem 6

In the $\{|1\rangle, |2\rangle, |3\rangle\}$ orthonormal basis, the Hamiltonian and two other observables are given by

$$\hat{H} = \hbar\omega_0 \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \hat{A} = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \hat{B} = b \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix},$$

and the state of the system at time $t = 0$ is given by

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{\mathrm{i}}{\sqrt{2}}|2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \mathrm{i} \\ 0 \end{pmatrix}.$$

Note that $\hat{H}$ commutes with $\hat{A}$, but not with $\hat{B}$. The eigenvalues of $\hat{B}$ are $2b$ and 0, and its eigenstates are

$$\text{eigenvalue } 2b: \quad |b_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},$$

$$\text{eigenvalue } 0: \quad |b_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad \text{and} \quad |b_3\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

(a) At $t = 0$, find $\langle A \rangle$ and ΔA.

(b) Write the state at an arbitrary later time, $|\psi(t)\rangle$.

(c) Find $\langle B \rangle(t)$, the expectation value of B at arbitrary time t.

(d) If B is measured at $t = 0$, what are the possible results and their probabilities?