# Department of Physics

# Preliminary Exam January 2–6, 2007

## Day 4: Thermodynamics and Statistical Physics

## Saturday, January 6, 2007

## 9:00 a.m. – 12:00 p.m.

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**Instructions:**

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Dishonesty and Academic Misconduct will be followed.

**Physical Constants and Conversions for Exam**:

$$R = N_A k_B = 8.31\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1} \qquad N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$$

$$1\,\text{atm} = 101\,\text{kPa} \qquad 1\,\text{liter}\cdot\text{atm} = 101\,\text{J}$$

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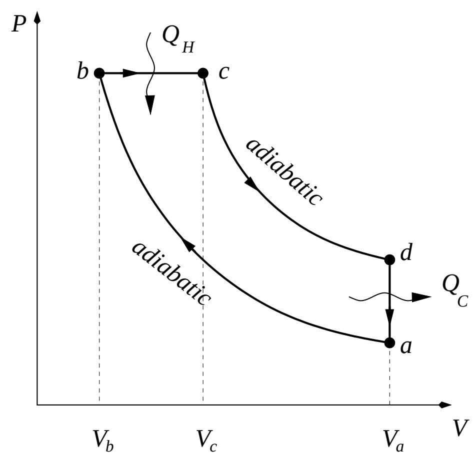
## Problem 1

**Diesel Engine:** The operation of a Diesel engine can be idealized by the cycle shown in the figure. Air is drawn into the cylinder during the intake stroke (not shown in the figure). The air is compressed adiabatically, path *ab*. At point *b*, diesel fuel is injected into the cylinder which immediately burns since the temperature is very high. Combustion is slow, unlike a gasoline engine, and during the first part of the power stroke, the gas expands at (nearly) constant pressure, path *bc*. After turning, the rest of the power stroke is adiabatic, path *cd*. Path *da* corresponds to a rejection of heat just prior to the exhaust stroke (also not shown in the figure).




(a) Show that, for a quasistatic reversible engine undergoing this cycle using an ideal gas, the ideal efficiency $e$ is given by:

$$e = 1 - \left(\frac{1}{\gamma}\right) \frac{(V_a/V_c)^{-\gamma} - (V_a/V_b)^{-\gamma}}{(V_a/V_c)^{-1} - (V_a/V_b)^{-1}}\,,$$

where $V_a/V_b$ gives the compression ratio, $V_a/V_c$ gives the expansion ratio, and $\gamma \equiv C_P/C_V$.

(b) If $V_a/V_b = 15.0$ and $V_a/V_c = 5.0$, calculate the efficiency assuming the gas is diatomic (like $O_2$) and ideal.

**Problem 2 Application of Principles to a Non-Ideal Gas:**

(a) Prove with the aid of Maxwell's equation $\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$ that in general

$$\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P .$$

For the rest of this problem, consider a non-ideal gas whose equation of state is $Pv = RT + AP$, where $A$ is a function of $T$ only.

(b) Using the result from part (a), show that

$$c_P = c_{P0} - PT\frac{\partial^2 A}{\partial T^2} ,$$

where $c_{P0}$ is the value of $c_P$ at $P = 0$, which is the value for an ideal gas.

(c) Given that, for a certain monatomic non-ideal gas, $A = b - a/RT$ with $a$ and $b$ constants, calculate the value of $c_P$ at $P = 5\,\mathrm{atm}$ and $300\,\mathrm{K}$ where $a = 1.40\,\mathrm{atm\,liter^2\,mol^{-2}}$.

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**Problem 3**

**Formal Results:** Derive the following well known thermodynamic identity for the case of a general hydrostatic thermodynamic system:

$$C_P - C_V = TV\beta^2/\kappa_T ,$$

where $C_P$ and $C_V$ are the heat capacities at constant pressure and constant volume, respectively, and

$$\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \qquad \kappa_T \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

HINT—If you don't know where to begin, begin by regarding $S=S(T,V)$, then write out $\mathrm{d}S$ appropriately.

## Problem 4

**Quantum Statistics:** The revolutionary concept of energy quantization, introduced by M. Planck in 1900 during his studies of blackbody radiation, signifies the birth of quantum theory and modern physics. Answer the following questions:

(a) Are photons *fermions* or *bosons*, and *why*?

(b) What is the fundamental difference between the photon statistics and general Bose-Einstein statistics?

(c) Denote by $n_r$ the number of photons in the quantum state with energy $\epsilon_r$. Using the partition function, derive an expression for the average number $\bar{n}_r$ as a function of temperature. Analyze and discuss the low temperature limit and high temperature limit of $\bar{n}_r$.

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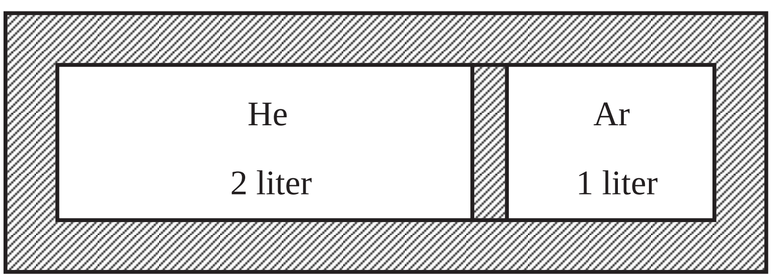
## Problem 5

**Entropy of Ideal Gas System:** A thermally insulated cylinder is partitioned into 2 liter and 1 liter sections by a fixed, insulating wall (see figure). The left compartment is filled with He gas at 1 atm of pressure and 280 K while the right compartment is filled with Ar gas at 3 atm of pressure and 320 K. Answer the following questions (all gases may be treated as ideal):




(a) What are the internal energies of the He and Ar gases, respectively?

Now the partition is suddenly removed. The two gases are allowed to mix and reach a new thermodynamic equilibrium.

(b) What is the final temperature of the system? (The heat capacity of the cylinder can be ignored.)

(c) What is the change of entropy of the system (He + Ar gases) caused by the mixing?