Department of Physics

Preliminary Exam January 2–5, 2008

Day 1: Classical Mechanics

Wednesday, January 2, 2008

9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

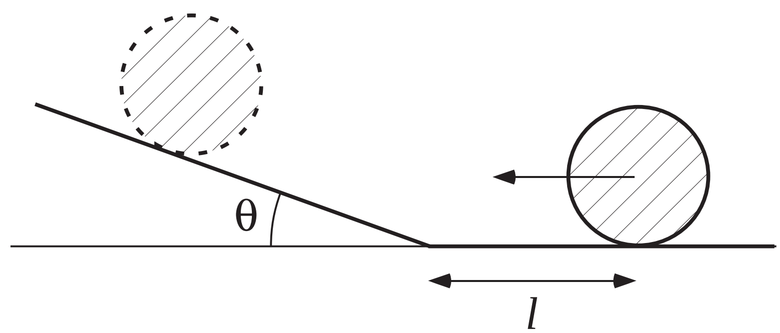
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

There are five problems. Answer all five. Each problem carries 20 points. Include all relevant intermediate steps.

Problem 1



Two cylinders are given, both of mass $m = 50.0\,\text{kg}$ and radius $R = 0.400\,\text{m}$. One cylinder (h) is hollow with all mass a distance R from its axis, whereas the other cylinder (s) is solid and of uniform density. At time $t = 0$ each cylinder is rolling on a horizontal surface with speed $v = 5.00\,\text{m/s}$ towards an incline with slope $\theta = 20.0°$, which begins $l = 6.00\,\text{m}$ further on (see figure above). Assuming the cylinders roll without slipping and without losses of energy due to friction, answer the following questions for each of the two cylinders:

(a) What is its moment of inertia I_i, $(i = \text{h}, \text{s})$, about its symmetry axis? Derive your results.

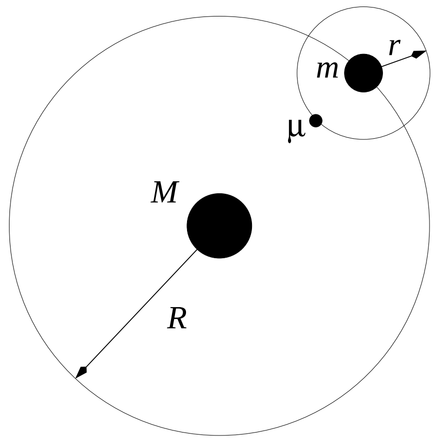
(b) How far up the incline will each cylinder travel? Give your answer as the distance d_i along the surface of the incline.

(c) How long does it take each cylinder to return to its starting point at $t = 0$?

(d) How big an average force is needed to stop each cylinder after its return to the starting point, assuming the force acts over an additional distance of $D = 2.20\,\text{m}$?

Problem 2

The gravitional sphere of influence of a planet. Consider a planet (mass m) and its small moon (mass μ) orbiting a star (mass M), as illustrated in this top view.



For a small-enough moon orbit, the moon and planet are effectively a two-body system. However, beyond some critical orbital radius r_H from the planet, called the Hill radius, the moon is no longer firmly gravitationally "bound" to the planet. Instead, the star's influence will make the moon susceptible to being "removed" from the planet through the influences of the star and any other planets in the vicinity.

Work through steps (a)–(e) below to show that the Hill radius r_H can be approximated as

$$r_H = \left(\frac{m}{3M}\right)^{1/3} R\,.$$

Assume all orbits are circular, and consider the specific geometry in which the moon is located between the centers of the star and planet.

(a) Write a general expression for the magnitude of the acceleration due to Newtonian gravity of an object located an arbitrary distance d from the star.

(b) Write a general expression for the magnitude of the centrifugal acceleration experienced by an object circling the star in terms of its orbital distance d and steady angular speed ω.

(c) Apply the results from parts (a) and (b) to find an expression for the angular velocity of an object in circular orbit around the star in terms G, M, and d.

(d) Observe that the gravitationally bound moon has an average angular speed around the star that equals the planet's, and therefore lower than that which it would have were it alone in its orbit. This is possible because of the planet's gravity. Use these considerations to write an equation relating the gravitational accelerations of the moon due to the planet and the star, and the centrifugal acceleration of the moon around the star at the distance r_H from the planet.

(e) Express the foregoing result in terms of the ratio $r_H/R \ll 1$. Then, keeping only zero- and first-order terms in that ratio, and using simple analytic approximations, obtain the requested expression for r_H.

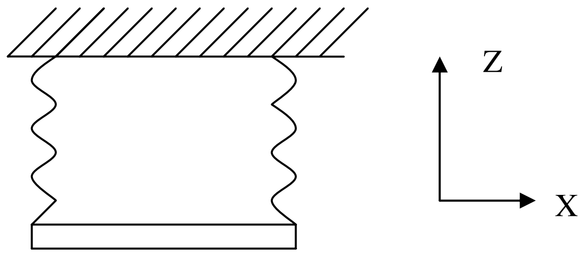
Problem 3

A spring pendulum consists of a point mass m attached to one end of a massless spring of spring constant k and unstressed length l obeying Hooke's law. The other end of the spring is pivoted and the system moves in a vertical plane.

(a) Obtain the Lagrangian for the system in terms of suitable generalized coordinates. Write down the Lagrange's equations of motion.

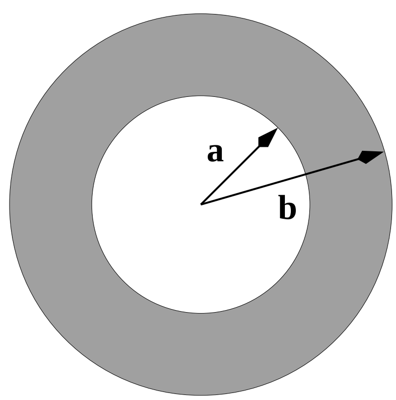
(b) Construct the Hamiltonian for the system. Show that the Hamilton's equations contain the same information as the Lagrange's equations.

Problem 4



A rigid, uniform, thin bar of mass M and length L is attached to two identical massless springs (each of force constant k) as shown in the figure. Assume the motion is constrained to the x-z plane and the center of mass moves only vertically along the z–axis direction. Find the normal frequencies of small oscillations of the system about its equilibrium position. Express the frequencies in terms of k, M, and L.

Problem 5



A thin, uniform disk has a circular hole cut out from its center as shown in the accompanying figure. The radius of the disk is b, while that of the hole is a, and the resulting object has a total mass M.

(a) Obtain the moment of inertia tensor of the object about the center of mass. What are the principal moments and principal axes?

(b) If the object rotates about an axis passing through a point on the outer edge of the disk and perpendicular to the plane of the disk with an angular speed ω, what is its kinetic energy?