Department of Physics

Preliminary Exam January 2–5, 2008

Day 2: Electricity, Magnetism and Optics

Thursday, January 3, 2008
9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each of questions 2–5, as well as the separate parts of question 1, on a separate sheet of paper.

   If more than one sheet is required, staple all corresponding pages together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. Each question carries the amount of credit indicated; they are *not* weighted equally. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Additional instructions and information:
There are five problems; the point value of each problem is given, and they add up to 100. Each problem should be started on a separate sheet. *In addition, the point value of each part of Problem 1 is given, and each part of that problem should be done on a separate sheet. Do not staple the parts of Problem 1 together.*
The problems are written using the SI system of units; if you prefer to use the cgs-Gaussian system, you may simply replace $4\pi\epsilon_0$ by 1 in the relevant equations given in the problems.

Problem 1

(25 points) The parts of this problem are independent short problems that do not require long answers. *Work each part on a separate sheet* and *do not staple different parts together.*

(a) (7 points) Consider two spherical conductors of different diameters; the two spheres are held at fixed positions very far apart from one another, and initially, both are uncharged. Then, the first sphere is charged to a total charge Q and brought into electrical contact with the second sphere by means of a negligibly thin conducting wire which is subsequently removed; as a result, there is now a charge q on the second sphere. The first sphere is then recharged to Q and again brought into electrical contact with the second, and this process is repeated *ad infinitum.* What is the final charge on the second sphere?

(b) (6 points) Consider a planar interface between a positive-refractive-index medium ($n_1 = 1$) and a negative-index material ($n_2 < 0$). If an object is placed a distance L from the surface on the $n_1 = 1$ side, find the location of the image that is formed on the $n_2 < 0$ side.

(c) (6 points) We can compute the total energy U of a charged particle by using the point charge model of a uniform sphere of radius a and its associated charge q uniformly distributed on the surface. Knowing that the magnitude of the electric field (of a point charge) is $q/4\pi\epsilon_0 r^2$, and that the energy density is

$$u = \frac{\epsilon_0 E^2}{2} = \frac{q^2}{32\pi^2\epsilon_0 r^4},$$

the total energy can be calculated by integrating over all space (with the lower limit of integration variable r as a and the upper limit as ∞):

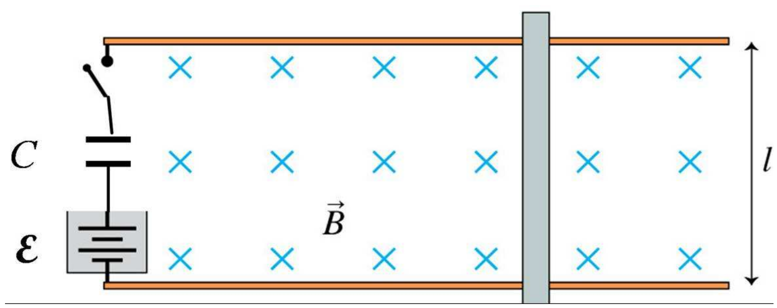
$$U = \frac{1}{2}\frac{q^2}{4\pi\epsilon_0}\frac{1}{a}.$$

Here's the problem: All is well with this calculation until we set $a = 0$ for a point charge. Note that because the energy density of the field varies inversely as the fourth power of the distance from the center, its volume integral is infinite. Is this acceptable? That is, can there be any observable consequences of having an infinite amount of energy in the field surrounding a point charge?

(d) (6 points) The beam from a diffraction limited He:Ne laser has a wavelength of 632 nm, a peak intensity of $10\,\mathrm{W/cm^2}$, and can be considered to have a diameter of 1 mm as it leaves the laser. The laser is aimed at a mirror placed on the moon by the Apollo astronauts. Estimate the peak intensity of the beam as it strikes the mirror. (The distance from the surface of the Earth to the surface of the Moon is approximately 3.76×10^5 km.)

Problem 2

(15 points) A bar of length l, mass m, and resistance R slides without friction on resistanceless rails in a uniform magnetic field. The rails are connected by a battery of emf $\mathcal{E}$ and a capacitor of capacitance C, as shown. The switch is closed at time $t = 0$, when the bar is at rest and there is no charge on the capacitor.



(a) Find the equation of motion of the bar.

(b) What is the final charge on the capacitor?

Problem 3

(20 points) Consider a uniform sphere of radius a with a constant volume charge density ρ and total charge Q.

(a) Using any method, show that the general expressions for potential at any point inside (ϕ_i) or outside (ϕ_o) the sphere are given by:

$$\phi_i(\mathbf{r}) = \frac{\rho}{6\epsilon_0}\left(3a^2 - r^2\right) = \frac{Q}{8\pi\epsilon_0 a}\left(3 - \frac{r^2}{a^2}\right)$$

and

$$\phi_o(\mathbf{r}) = \frac{\rho a^3}{3\epsilon_0 r} = \frac{Q}{4\pi\epsilon_0 r}\,,$$

respectively.

(b) Calculate the electric fields inside, $\mathbf{E}_i$, and outside, $\mathbf{E}_o$, using the results of part (a) (or otherwise).

(c) Calculate the total potential energy of this system using the results of part (a). Express your final answer in terms of the total charge Q.

(d) Calculate the energy density for outside and inside the sphere. Show that the total potential energy calculated from the energy density is the same as found in part (c).

(e) What fraction of the total energy in part (c) is now regarded as being outside of the sphere?

Problem 4

(20 points) Apart from its many other uses, a Michelson interferometer can be employed as a spectrometer. To see how this comes about we will initially consider a light source with a very simple spectrum, a plane wave with wavenumber k_0. This light is incident on the beam splitter of the Michelson interferometer and the interferometer has a path length difference of d between its two arms.

(a) Calculate the intensity at the output of the interferometer as a function of d.

(b) If instead the input light consists of multiple plane waves with different wavenumbers, the intensity can be calculated by adding the intensities of each of the individual plane waves together. Why does this work (i.e., why doesn't the light from one plane wave seem to interfere with light from another plane wave)?

(c) Using the results from parts (a) and (b), show that the intensity at the interferometer's output for a source with a uniform distribution of wavenumbers between $k_1 = k_0 - \frac{\Delta k}{2}$ and $k_2 = k_0 + \frac{\Delta k}{2}$ with a total intensity of I_0 is

$$I(d) = I_0 \left[1 + \operatorname{sinc}\left(\frac{\Delta k\, d}{2}\right) \cos k_0 d\right] .$$

(Note: $\operatorname{sinc} x \equiv \frac{\sin x}{x}$.)

(d) If the source considered in part (c) consists of white light (wavelength range 400 to 700 nm), approximately how many "white light" fringes will be visible as the path length difference is scanned through $d = 0$? It may help to sketch the function given in part (c).

Problem 5

(20 points) A ferro-fluid is a super-paramagnetic ($\mu/\mu_0 \gg 1$) liquid that becomes magnetized in an external magnetic field. Consider the following experiment: A drop of ferro-fluid is placed on the surface of water in a glass. It immediately spreads to form a uniform thin film on the water surface. A linear bar magnet, vertically oriented, is brought down near the ferro-fluid film. Remarkably, the film moves on the surface but away from the magnet to leave a clear patch on the surface (see Fig. 1). The purpose of this problem is to begin to explain this observation by following the hint below.

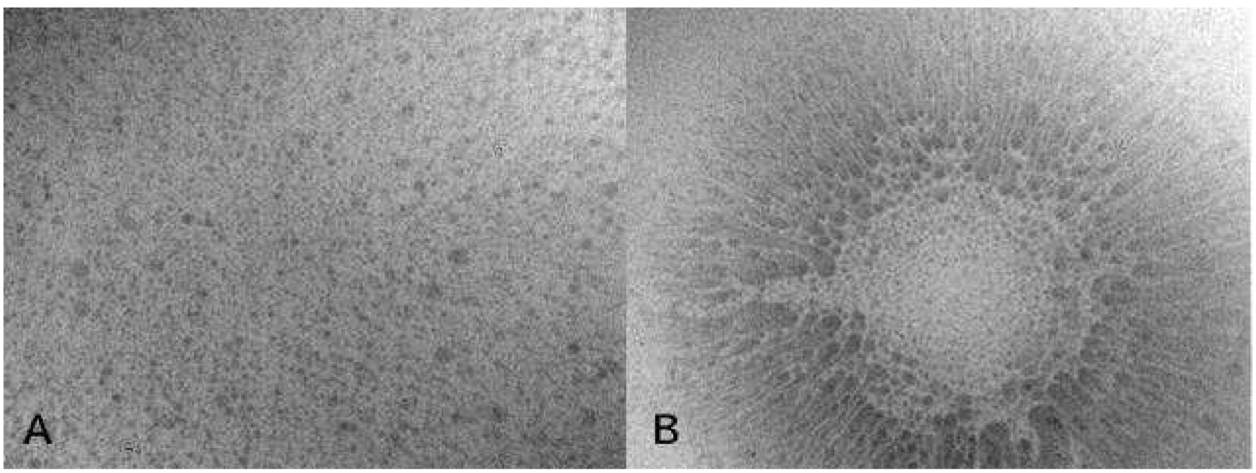


Fig. 1. Image A shows the ferro-fluid film uniformly spread on the surface. Image B shows the film clearing from beneath the bar magnet and also coming in from the outer edges of the surface to eventually form a ring of ferro-fluid on the water surface.

Hint: Consider the experimental geometry shown in Fig. 2. Assume the bar magnet (dipole moment $\mathbf{m}$) produces a dipole field as given by:

$$\mathbf{B}_0 = \frac{\mu_0}{4\pi r^3}\left(3\left(\mathbf{m}\cdot\hat{\mathbf{r}}\right)\hat{\mathbf{r}} - \mathbf{m}\right) = \frac{\mu_0 m}{4\pi}\left[\frac{3\rho z}{\left(\rho^2+z^2\right)^{5/2}}\hat{\boldsymbol{\rho}} + \frac{2z^2-\rho^2}{\left(\rho^2+z^2\right)^{5/2}}\hat{\mathbf{z}}\right]$$

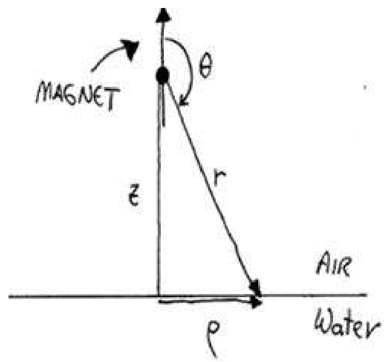


Fig. 2. Sketch of the experimental setup, showing the magnetic dipole moment and its relation to the water surface.

Calculate the energy density within the (initially) uniform thin film as a function of the radial surface distance away from the axis of the bar magnet. If the energy density changes as a function of ρ, there will be a body force on the film, just as on particles pulled to the lowest potential energy in a gravitational field.