Department of Physics

Preliminary Exam January 2–5, 2008

Day 3: Quantum Mechanics and Modern Physics

Friday, January 4, 2008

9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Problem 1

Two copper conducting wires are separated by an insulating copper oxide (CuO) layer. The oxide layer can be modeled by treating it as a square potential barrier of height 10.0 eV (the CuO band gap). Determine the transmission probability T for a beam of electrons with kinetic energy of 7.0 eV and a layer thickness of 1.0 nm.

Problem 2

A white dwarf is a star which has used up its nuclear fuel and has contracted under its own weight to a state of high density and very low temperatures. It is held up by the kinetic energy of its degenerate electrons. Assume that all electrons in the star behave like free electrons in a metal, that all the electron states are full up to the Fermi energy, that the electrons are non-relativistic, and that the star is composed of helium. First, derive an expression for the total energy of the dwarf star. Then, show that minimizing the total energy with respect to the radius leads to an equilibrium radius of the star given by

$$R = \frac{C\hbar^2}{G m_e m_p^{5/3} M^{1/3}}$$

where C is a numerical constant, G is the gravitational constant, m_e and m_p are the electron and proton masses, and M is the mass of the star.

Problem 3

A sample consisting of 1 g of pure ^{226}Ra is sealed in a glass ampoule. ^{226}Ra has a half-life of 1602 years and decays to ^{222}Rn by α-particle emission. ^{222}Rn has a half-life of 3.82 days and decays by α-particle emission into ^{218}Po. Calculate the number of disintegrations per second of ^{222}Rn in the ampoule after a period of one hour.

Problem 4

Consider a three-dimensional, spherically symmetric analog of the infinite square well that satisfies the following Hamiltonian:

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(r)$$

$$V(r) = \begin{cases} \infty, & r < a \\ 0, & a \leq r \leq b \\ \infty, & r > b \end{cases}$$

Calculate the energies of the spherically-symmetric eigenfunctions that have angular momentum of zero. First, what are the wave functions in the three regions of space? Secondly, what are the boundary conditions that must be satisfied? Finally, what energy eigenvalues satisfy these boundary conditions?

Problem 5

A particle of mass m is initially in the lowest ($n = 1$) state of a one-dimensional infinite square well extending from $x = 0$ to $x = L/2$. Then, the right-hand wall of the well is moved instantaneously to $x = L$. Calculate the probability of the particle being found in the following quantum states *of the widened well:*

(a) the ground state ($n = 1$);

(b) in the first excited state ($n = 2$).

Useful integral: $\int_0^\pi \sin^2 x \, dx = \frac{\pi}{2}$; **useful trig relation:** $\sin 2x = 2 \sin x \cos x$.