Department of Physics

Preliminary Exam January 2–5, 2008

Day 4: Thermodynamics and Statistical Physics

Saturday, January 5, 2008

9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Problem 1

(a) Derive an expression for $dG(T, P, N)$ by Legendre transforming $U(S, V, N)$ and write an expression for the chemical potential μ.

(b) Determine the pressure dependence of μ, starting with

$$\left(\frac{\partial \mu}{\partial P}\right) = \frac{\partial}{\partial P}\left(\frac{G}{N}\right) .$$

Problem 2

Three particles at the corners of an equilateral triangle each carry a quantum-mechanical spin $\frac{1}{2}$, and their mutual spin Hamiltonian is given by

$$H = \frac{\lambda}{3}\left(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3 + \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3\right) .$$

List the energy levels of this spin system, giving their total spin values and degeneracies. Deduce the partition function Z.

Problem 3

The density of the electron states in a metal is $g(E) = AE^{1/2}$, where A is a constant and E is measured from the bottom of the conduction band.

(a) Show that the total number of states occupied in the conduction band is

$$\frac{2}{3}A(E_F)^{3/2} \quad \text{at } T = 0 \text{ K}.$$

(b) About what fraction of the conduction electrons are within kT of the Fermi energy assuming $kT \ll E_F$?

Problem 4

Consider a design for a 40-W engine operating in conjunction with two heat reservoirs, one maintained at the fixed temperatures of 600 K and the other maintained at 400 K. The engine extracts heat from the hot reservoir at a rate of 45 J/s, and rejects heat to a cold reservoir at a rate of 5 J/s.

(a) Discuss the possibility/impossibility of this engine.

(b) If this engine is feasible, calculate its efficiency and compare it with the efficiency of a Carnot engine. If the engine is not feasible, calculate the maximum amount of work that can (in principle) be obtained for every 100 J of heat extracted from the hot reservoir.

(c) Consider now a more complicated problem: two systems with identical number of moles n and constant specific heat capacities c_V are kept at constant volume, so no work can be done on or by either of the systems. The internal energy of each system is related to its temperature by the equation $U = nc_V T$. The initial temperature of one of the systems is T_1 and of the other is T_2. An engine is to deliver work to a purely mechanical system drawing energy from the two thermodynamic systems. What is the maximum work that can be so delivered?

Problem 5

Consider n moles of an ideal gas system enclosed in a volume V.

(a) Consider the thermodynamic equation $dU = dQ - dW$. Discuss whether the signs of the terms dQ and dW are correct or not. Explain your reasoning.

(b) Discuss the assumptions that make this gas ideal. How does an ideal gas differ from a van der Waals gas? (Note: since the equations of state of both gases are provided below, it is not sufficient merely to reproduce the equations: you have to explain the assumptions and why the equation for the van der Waals gas differs from the ideal gas equation of state.)

$$\textbf{Ideal: } P = nRT/V \qquad \textbf{van der Waals: } P = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

(c) Two similar systems, one containing an ideal gas and one containing a van der Waals gas, are initially at pressure P_0, volume V_0, and temperature T_0, where T_0 is below the critical temperature of the van der Waals gas. Discuss the possible outcomes and the behavior of the isothermal compressibility κ_T in both cases as the systems are compressed isothermally.

(d) An automobile tire is inflated to a pressure of 270 kPa at the beginning of a trip. After three hours of high-speed driving the pressure is 300 kPa. What is the internal energy change of the air in the tire between pressure measurements? Assume the air is an ideal gas with a constant specific heat capacity $c_V = 5R/2$ and that the internal volume of the tire remains constant at $0.057\,\mathrm{m}^3$.