# Department of Physics

## Preliminary Exam January 5-9, 2009

## Day 1: Classical Mechanics

## Monday, January 5, 2009

## 9:00 a.m. – 12:00 p.m.

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### Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all the problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. This exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a deviation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

## Problem 1

### The spin-down luminosity of a pulsar

A pulsar is the compact remnant of a large star that has exploded. As a result of its intense magnetic field and rapid spin, its kinetic energy of rotation is gradually converted into light. One well-studied pulsar has radius $R$ = 15 km, mass $M = 4 \times 10^{30}$ kg, and rotation period $T$ = 0.033 s. This spin period is slowly, steadily increasing at the rate $dT/dt$ = +10 μs/yr.

As outlined below, compute the luminosity (brightness) of the pulsar in J/s assuming that its rotational kinetic energy is being converted totally into light :

(a) Use cylindrical coordinates to derive an expression for the moment of inertia $I$ of this pulsar in terms of its mass $M$ and radius $R$. Assume that it is spherical, and that its density is uniform because it consists almost entirely of neutrons in a degenerate state.

(b) Express the angular frequency $\omega$ of the pulsar in terms of its spin period $T$.

(c) What is the rotational kinetic energy $K$ of the pulsar in terms of $M$, $R$, and $T$?

(d) Derive an expression for $dK/dt$, given that the pulsar's mass and radius remain constant.

(e) Use the result from (d) to estimate the numerical value of the pulsar's luminosity in J/s.

## Problem 2

A pendulum clock is mounted on a horizontal rotating turntable, with the point of suspension above the center of the turntable. When the turntable is at rest, the pendulum oscillates at small amplitudes with a period of 1 second. The pendulum always swings in the same plane with respect to the clock, which rotates with the turntable.

(a) Write down the Lagrangian for the motion of the pendulum bob, and determine the pendulum period as a function of the turntable angular velocity ω in the limit of small amplitude oscillations.

(b) Assuming small oscillation amplitudes, determine the slowest turntable rotational speed at which the pendulum ceases to oscillate about the lowest position in its path.

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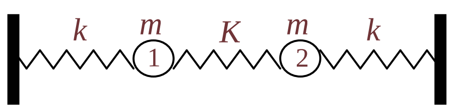
## Problem 3

(a) Two identical harmonic oscillators, each consisting of a mass *m* connected to a rigid wall by a spring with spring constant *k*, are coupled by another spring with spring constant *K*, as shown in the sketch. Find the frequencies of oscillation of the normal modes of the coupled system, $\omega_+$ and $\omega_-$, and show that $\omega_- < \omega_0 < \omega_+$, where $\omega_0$ is the oscillation frequency of one mass when the other is held stationary.




(b) Now mass 1 is driven and damped; the driving force is $Fe^{-i\omega t}$, and the damping force, proportional to velocity, is $-m\gamma\dot{x}_1$. Mass 2 is undamped, and is driven only by its coupling to mass 1. Find an expression for the displacement of mass 1, and show that when the driving frequency $\omega$ is equal to $\omega_0$ (and $K \neq 0$) this displacement vanishes ($x_1(t) = 0$). (This zero amplitude, at the very frequency where the resonant maximum amplitude would be if mass 2 were held stationary, is an exact classical analog to the phenomenon in optical physics known as electromagnetically induced transparency.)

## Problem 4

An engine of mass $M$ generates a constant driving force $F$ working against a drag resistance proportional to the square of its speed. The maximum speed it can attain is given by $U$. Assume that the engine starts from rest.

(a) How much time does the engine take to reach half the maximum speed?

(b) How much distance is covered in this time?

## Problem 5

### Coriolis force on a thrown object

A small object of mass $m$ is tossed vertically upward with initial speed $v_o$ from Earth's surface at a location in the northern hemisphere with colatitude $\theta$ (latitude = 90° – $\theta$). Due to Earth's rotation with angular speed $\omega$, the Coriolis force will cause the object to not land at its launching point. Recall that the Coriolis force is $\boldsymbol{F} = -2\mathrm{m}\ \boldsymbol{\omega} \times \boldsymbol{v}$.

(a) Neglecting air resistance, where will the landing point be relative to the launch point? (Just give the geographic direction.)

(b) Calculate the magnitude of $F$ in terms of a rectangular coordinate system centered at the launch point in which the coordinates ($x$, $y$, $z$) correspond to (east, north, vertical), as shown. In this coordinate system, $\omega$ will have both $y$ and $z$ components.

(c) Show that the object will land a distance $d$ away from its starting point given by $d = \dfrac{4\omega \sin\theta\, v_0^3}{3g^2}$, where $g$ is the acceleration due to gravity at the surface.