

# Department of Physics

## Preliminary Exam January 5-9, 2009

### Day 2: Electricity, Magnetism and Optics

Tuesday, January 6, 2009

9:00 a.m. – 12:00 p.m.

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#### Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all the problems together. This exam has *five* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. This exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a deviation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

## Problem 1

Consider the two infinitely long straight currents shown in Fig. 1 below. Current  $I'$  coincides with the  $y$ -axis. Current  $I$  is parallel to the  $yz$ -plane, is at a distance  $\rho$  from it, crosses the  $x$ -axis at  $y = z = 0$ , and makes the angle  $\alpha$  with the  $xy$  plane as shown. Calculate the force on current  $I$  of circuit  $C$  due to current  $I'$  of circuit  $C'$ . Use rectangular coordinates and relate your calculations to angle  $\alpha$ .

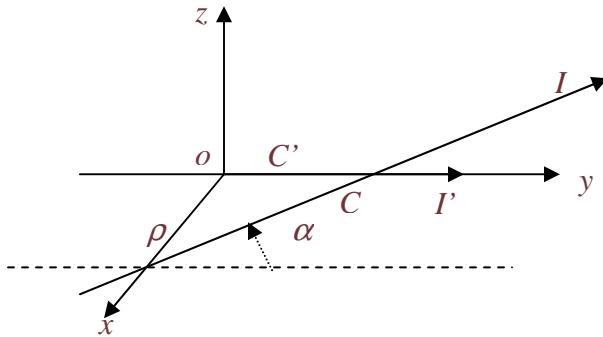


Figure 1

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## Problem 2

The beamsplitter in a certain Michelson interferometer consists of a 2 mm thick glass microscope slide, one side of which is coated with aluminum to make a partially reflecting mirror. The beamsplitter makes an angle of 45 degrees to an incoming plane wave and has an index of refraction of  $n_r = 1.4$  for 650 nm light, and  $n_b = 1.5$  for 450 nm light. The interferometer is set up so that the optical path length (OPL) difference between the two arms is zero when the 650 nm light is used. This particular interferometer does not have a compensating plate.

- Draw a diagram of the interferometer described above.
- Find the OPL associated with the glass for each of the two arms with 650 nm light.
- If blue light at 450 nm is used instead, the OPL difference between the two arms is no longer zero. Determine its new value.

- (d) How far will one of the mirrors need to be moved to regain zero OPL difference for the blue light?
- (e) Still using the blue light, how many dark fringes will move through an observer's field of view as the mirror is moved by the amount you calculated in (d)?
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### Problem 3

A conductor has the shape of an infinite conducting plane except for a hemispherical boss of radius  $a$ . A charge  $q$  is placed above the center of the boss at a distance  $p$  from the plane. Calculate the force on the charge.

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### Problem 4

A point charge  $q$  is located at the center of a dielectric sphere of radius  $a$  and dielectric constant  $k_e$ .

- (a) Find  $\vec{D}$ ,  $\vec{E}$ , and  $\vec{P}$  everywhere and plot your results.
- (b) What is the total bound charge on the surface of the sphere?
- (c) Are your answers in part (a) consistent with the boundary conditions? Explain.
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### Problem 5

It is well known that an aperture used to change the spatial *intensity* profile of a light wave produces diffraction in the focal plane of a lens (e.g. diffraction from a single slit). Less commonly considered is that “phase” apertures (which change the *phase* profile of a wave) also produce diffraction. In this problem we will look at what can

happen when these two types of apertures are combined. This problem is related to how information is read off a CD or DVD.

To begin with, take a plane wave of wavelength  $\lambda$  that is incident on a long slit of width  $w_{\text{slit}}$ . Behind the slit is a lens of focal length  $f$ . Only light that passes through the slit travels through the lens. The effect of the slit may be defined via an aperture function

$$h(w, x) = \begin{cases} 1, & -\frac{w}{2} < x < \frac{w}{2} \\ 0, & \text{otherwise} \end{cases} \quad \text{where } x \text{ is the spatial coordinate across the slit.}$$

- (a) Show that the functional form of the *intensity* pattern observed on a screen in the focal plane of the lens is

$$I \propto w_{\text{slit}}^2 \frac{\sin^2\left(\frac{kw_{\text{slit}}}{2f} X\right)}{\left(\frac{kw_{\text{slit}}}{2f} X\right)^2} = w_{\text{slit}}^2 \text{sinc}^2\left(\frac{kw_{\text{slit}}}{2f} X\right),$$

where the coordinate on the screen is  $X$  and  $k = 2\pi/\lambda$ . You may ignore any constants of proportionality and the diffraction produced by the lens.

Next, a piece of plastic of width  $w_{\text{plas}}$  is placed in the center of the slit. The plastic changes the phase of any light passing through it by  $\pi$  radians.

- (b) Provide a *justification* as to why it is possible to write an aperture function for the slit/plastic combination as  $f(x) = h(w_{\text{slit}}, x) - 2h(w_{\text{plas}}, x)$ .
- (c) Using the result in (a) together with the aperture function of (b), find the intensity pattern in the focal plane of the lens. You may again ignore constants of proportionality.
- (d) Determine the relationship between  $w_{\text{slit}}$  and  $w_{\text{plas}}$  when the intensity on the screen behind the lens is zero at  $X = 0$ .