# Department of Physics

# Preliminary Exam January 5–9, 2009

## Day 3: Quantum Mechanics and Modern Physics

## Thursday, January 8, 2009

## 9:00 a.m.–12:00 p.m.

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**Instructions:**

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

**Special Instructions:** There are five problems. Attempt all five. Each problem carries 20 points. Make sure that you include all relevant intermediate steps. The following integral may be useful:

$$\int_{-\infty}^{\infty} e^{-\alpha x^2}\, dx = \sqrt{\frac{\pi}{\alpha}}$$

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## Problem 1

(a) Two inertial reference frames $S$ and $S'$ are in relative motion with a speed $v$. Two clocks located at the origin of $S$ and $S'$ are synchronized when the origins of the systems coincide. After a time $t$, an observer at the origin of the $S$ system observes the $S'$ clock by means of a telescope. What would the $S'$ clock read to the $S$ observer?

(b) If a meter stick is fixed at an angle $\theta$ from the axis of relative motion in the $S'$ system, what is the length and orientation of the stick as measured by the observer in $S$?

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## Problem 2

The Hamiltonian for a particle of mass $m$ confined to one dimension and subject to a harmonic potential is given by

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2\,.$$

(a) Sketch qualitatively the wave function $\psi(x)$ for the ground state and for the first excited state of the system.

(b) What will be the most probable values that will result from measurements of position and momentum of the particle in the ground state and in the first excited state? Explain your reasoning.

(c) Determine the uncertainty in the position and momentum measurements in the ground state of the system.

## Problem 3

Show that there is only one bound state in the one–dimensional attractive delta function potential $V = -v_0\delta(x)$. Find the energy and wave function of the state.

## Problem 4

Consider a one dimensional Hamiltonian

$$H = \frac{p^2}{2m} + V(x)$$

with eigenfunctions $\psi_k(x)$ and eigenvalues $E_k$.

(a) Calculate the commutator $[x,\ [x, H]]$.

(b) Using the results of part (a) show that

$$\sum_k (E_k - E_\ell)\,|x_{kl}|^2 = \frac{\hbar^2}{2m}$$

where

$$x_{kl} \equiv \int \psi_k^*(x)\ x\ \psi_\ell(x) dx$$

[**Hint:** You may consider taking expectation value of the result in state $\psi_\ell(x)$ in part (a).]

## Problem 5

Consider a three-dimensional system subject to a magnetic field $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}$. The interaction Hamiltonian can be written as

$$H_1 = -\mu \mathbf{B}\cdot\mathbf{J}$$

where $\mathbf{J}$ is the angular momentum operator. Find the correction to the unperturbed state $|j, m\rangle$ and its energy to the lowest non-vanishing order where the unperturbed Hamiltonian $H_0$ is given by

$$H_0 = \alpha J_z\ .$$

Perform the calculations for arbitrary value of $j$.