Department of Physics

Preliminary Exam January 5-9, 2009

Day 4: Thermodynamics and Statistical Physics

Friday, January 9, 2009

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all the problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. This exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a deviation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Special Instructions: There are a total of six problems in this exam. You only need to do 5 problems as follows: Problems 1-4, plus *either* Problem 5 *or* Problem 6.

You may need the following constants:

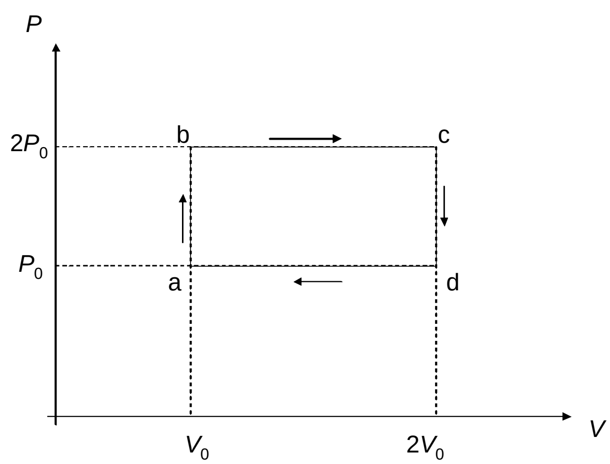
$$N_A = 6.023\times10^{23}\,\mathrm{mol}^{-1}$$
$$R = 8.314\,\mathrm{J}\cdot\mathrm{K}^{-1}\cdot\mathrm{mol}^{-1}$$

Problem 1

The following independent questions explore some basic concepts in thermodynamics.

(a) The molar specific heat c_P for aluminum (Al) varies approximately linearly with temperature from *24.4 J* mol^{-1} K^{-1} at *300 K*, to *28.1 J* mol^{-1} K^{-1} at *600 K*. Determine the heat required to increase the temperature of *2.5 moles* of Al from *300 K* to *500 K* at constant pressure.

(b) One mole of a monoatomic ideal gas is used in an engine that operates according to the cycle shown in the figure below. Calculate the efficiency η of such engine.



(c) Consider an ideal gas enclosed in a cylinder containing a piston that can be either fixed in place keeping the volume constant, or movable keeping the pressure constant. Explain qualitatively why C_P is larger than C_V.

Problem 2

The average pair of human lungs can hold about 6 L of air. Only 0.5 L of the air in lungs is exchanged during a normal breath.

(a) Calculate the number of oxygen molecules that one inhales during one normal breath (300 K and 1 atm pressure of dry air). The dry air contains 78.1% of nitrogen (N_2), 20.9 % of oxygen (O_2) and 1% of other trace chemicals. (1 atm = 1.013×10^5 N/m^2.)

(b) When the winter outdoor temperature dips into 260 K, how much heat is needed to warm up the inhaled air to body temperature (310 K) during 30 minutes of outdoor activity. Assuming that the normal breathing frequency is 10 times per minute, the heating is performed at constant pressure, and dry air is an ideal gas of diatomic molecules.

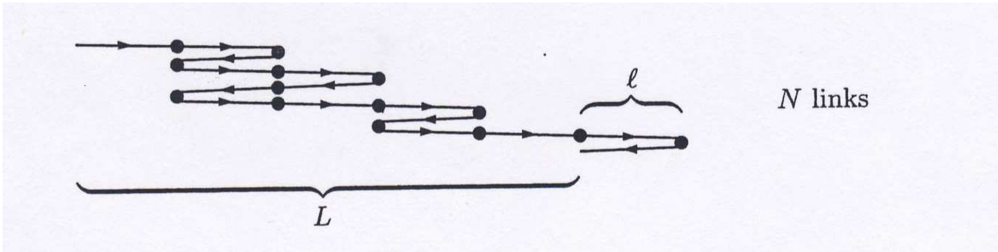
Problem 3

(Rubber band model) Polymers are made of very long molecules usually tangled up in a configuration. As a very crude model of a rubber band, consider the molecule as a chain of N links, each of length l (see the figure). Assume that each link has only two possible states, pointing either left or right. The total length L is the net displacement from the beginning of the first link to the end of the last link.

(a) Write down an expression for L in terms of N and N_R (N_R denotes the number of links pointing to the right).

(b) Find an expression for the entropy of this system in terms of N and N_R.
(Note: $\ln N! \approx N \ln N - N$ for $N >> 1$)

(c) Using a thermodynamic identity, you can express the tension force F in terms of a partial derivative of the entropy. From this expression, compute the tension F in terms of L, T, N, and l.



Problem 4

Consider a monoatomic, non-degenerate ideal gas system containing N_A atoms of gas A and N_B molecules of gas B in contact with a heat reservoir at temperature T (ignore spin states).

Using the properties of the canonical-ensemble partition function, show that the mean energy E, the entropy S, and the Helmholtz free energy F are additive (i.e., $\overline{E} = \overline{E}_A + \overline{E}_B$, $S = S_A + S_B$, $F = F_A + F_B$).

Considering gas A as a particle in a box with periodic boundary conditions, the quantized particle energies are $\varepsilon = \frac{2\pi^2\hbar^2}{m_A}\left(\frac{n_x^2}{L^2} + \frac{n_y^2}{L^2} + \frac{n_z^2}{L^2}\right)$, where n_x, n_y, and n_z are any set of integers (positive, negative, or zero). Show that the single-particle partition function for gas A is given by $\zeta_A = \frac{V}{h^3}(2\pi m_A kT)^{3/2}$. (You may use $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\pi/\alpha}$.)

Obtain an expression for the chemical potential of gas A as a function of the volume V, temperature T, and number of particles N_A.

Problem 5 (or do Problem 6)

Consider a free electron gas in a cube of volume $V = L^3$. The momentum is of the form $P_i = \pi\hbar n_i / L$, where $i = x, y, z$. Assume the electrons with energy >> $m_e c^2$, where m_e is the rest mass of the electron.

Show that in this relativistic limit the Fermi energy of a gas of N electrons is given by

$E_F = \hbar c \pi (3n/\pi)^{1/3}$ where $n = N/V$

Show that the total energy of the ground state of the gas is

$U_0 = 3NE_F / 4$

Problem 6 (or do Problem 5)

The high temperature behavior of iron can be summarized as follows:
α-iron is the stable phase for T < 900°C and this *same phase* is again stable for T > 1400°C (T_2); a second phase, called γ-iron, is stable at 900°C < T < 1400°C. The specific heats of each phase may taken as constant ($C_p^\alpha = 0.775\,\mathrm{J/g \cdot K}$ and $C_p^\gamma = 0.690\,\mathrm{J/g \cdot K}$). Assume that the process is isobaric (constant pressure) and that we have 1 g of iron. Let T_1 = 900°C, T_2 = 1400°C, and S_0 the entropy at T_1 in α-iron phase. You may use $\int \ln(ax)dx = x(\ln(ax) - 1)$.

Note: Even if you find this problem difficult, try to do as much reasoning as you can based on your physical intuition.

(a) Calculate the chemical potential μ and entropy S as a function of T, C_p^α, C_p^γ, and S_0.

(b) Qualitatively illustrate the entropy and the chemical potential as a function of temperature.

(c) Calculate the latent heat at each transition (at T_1 and T_2).