Department of Physics

Preliminary Exam January 4–8, 2010

Day 1: Classical Mechanics

Monday, January 4, 2010

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

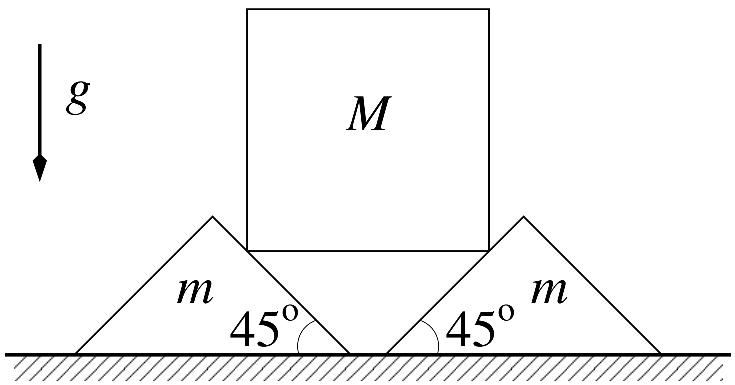
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

There are five problems. Answer all five. Each problem carries 20 points. Include all relevant intermediate steps.

Problem 1



Consider the figure above. The square block (mass M) is perched symmetrically atop two identical triangular blocks (each mass m). Friction is present but only between the triangular blocks and the floor below.

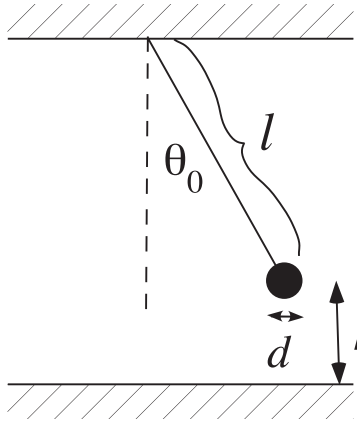
(a) What minimum value for the coefficient of static friction μ_s^{min} is needed to prevent the triangular blocks from slipping sideways? (Obtain μ_s^{min} algebraically, then numerically evaluate for the special case, $M = m$.)

For the remainder of this problem we shall assume a slightly modified scenario: At time $t = 0$, all friction is (somehow) suddenly removed, thereby allowing all of the blocks to slide freely (against the floor or each other).

(b) How much time does it take for the square block to hit the floor, given that its base is initially at a height h_0 above the floor? (Assume the block does *not* rotate as it falls.)

(c) Continuing the scenario of Part (b), at the instant before the square block hits the floor, what are the velocities (magnitude, direction) of each of the three blocks?

Problem 2



A pendulum consists of a spherical bob of mass m and diameter d suspended by a rigid, thin wire of length ℓ and negligible mass. The initial position of the bob is such that the wire makes an angle θ_0 with the vertical. The local acceleration due to gravity is $g = 9.8\,\mathrm{m/s^2}$. The bob is released from rest. Due to air resistance, it does not return exactly to its starting height h_0 after one full oscillation.

The force F of air resistance in this case is given by $F = cv^2$, where v is the speed of the bob and $c = CA\rho/2$, where $C = 0.4$ is the drag coefficient for a spherical object, A is its cross-sectional area, and ρ is the density of the air.

Estimate the vertical distance Δh between h_0 and the height of the bob h after one oscillation. To do this, complete steps (a)–(d) below:

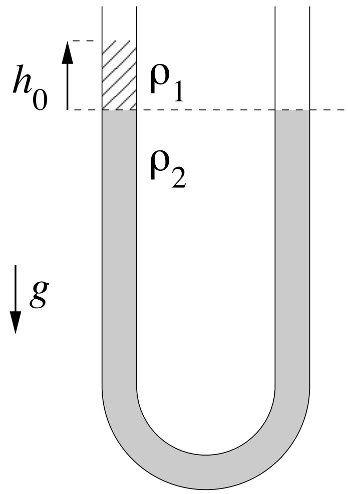
(a) Use energy conservation to express the velocity of the bob v in terms of the angle θ between the wire and the vertical at any time, and other parameters of the problem, neglecting air resistance.

(b) *Estimate* the work W done by air drag on the bob over the course of one full oscillation. To do this, first estimate $W_{1/4}$, the work done over the course of 1/4 of a full oscillation, in terms of c and other parameters of the problem.

(c) Use the work-energy theorem and result (b) to compute Δh.

(d) Finally, compute the numerical value of Δh for $\rho = 1.0\,\mathrm{kg/m^3}$, $d = 0.20\,\mathrm{m}$, $\ell = 5.0\,\mathrm{m}$, $m = 15\,\mathrm{kg}$, and $\theta_0 = 40.^\circ = 0.70\,\mathrm{radians}$.

Problem 3



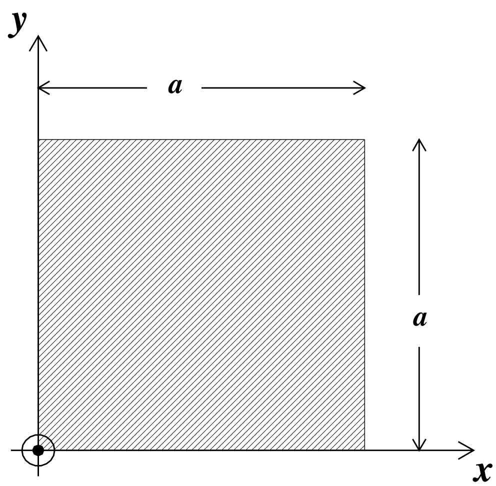
Consider two immiscible fluids of unequal densities ($\rho_1 < \rho_2$) in a fixed, U-shaped tube, open at each end. (For example, fluid #1 might be oil, and fluid #2, water.) The fluids are poured into the tube, then released from rest, starting from the initial geometry depicted in the accompanying figure.

(a) Find the difference in meniscus heights, $\Delta h \equiv h_{\text{left}} - h_{\text{right}}$, for the system in (mechanical) equilibrium. (Neglect surface tension effects.)

(b) Qualitatively interpret the result you calculated in Part (a). Does your result make good physical sense?

(c) Letting A denote the tube cross-sectional area, compute the change in gravitational potential energy $\Delta V \equiv V_{\text{f}} - V_{\text{i}}$, between the final (mechanical equilibrium) and initial (as depicted in figure) configurations of this system.

Problem 4



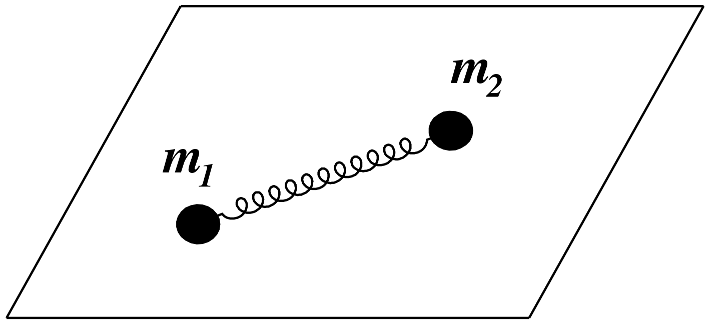
A thin, uniform, square sheet, having mass m and side a, lies in the x-y plane, as shown in the accompanying figure.

(a) Obtain the elements of the moment of inertia tensor, in terms of the coordinate system shown in the figure. Determine the principal moments† and the principal axes for this case.

(b) If the sheet is rotated about an axis along the sheet diagonal with a constant angular speed ω, what would be its kinetic energy? What is the angular momentum?

† You are only required to calculate the principal moments about the origin, not those about the center of mass.

Problem 5



A massless spring, of spring constant k and equilibrium length ℓ, connects two point masses, m_1 and m_2. Each mass is able to slide without friction in any direction along a smooth horizontal surface, as shown in the figure.

(a) Set up the Lagrangian for the system choosing an appropriate set of generalized coordinates. Obtain the Lagrange's equations of motion.

(b) Identify the cyclic coordinates for this system. What are the generalized momenta associated with each?

(c) Now, consider the special case where m_1 is *fixed* on the surface while m_2 is still allowed to move without friction along the surface. Obtain the Hamiltonian for the system in this case, and write down the Hamilton's equations of motion.