Department of Physics

Preliminary Exam January 4–8, 2010

Day 2: Electromagnetism and Optics

Tuesday, January 5, 2010

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. The questions carry *different weights* as indicated. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

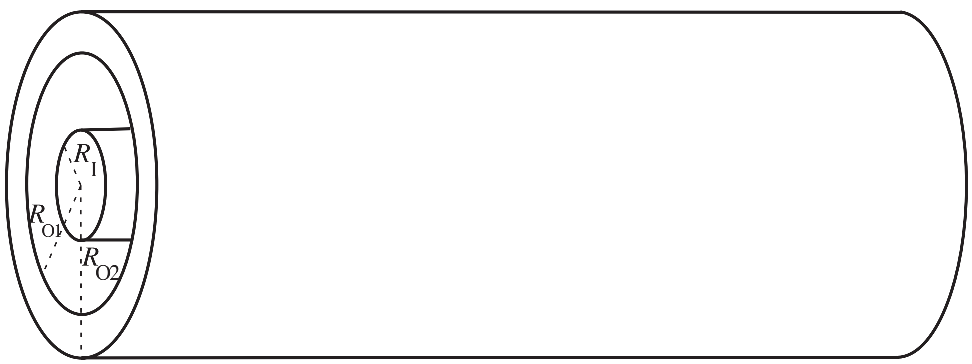
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Problem 1 (20 points)

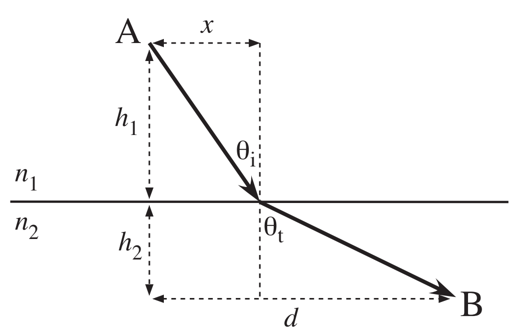
The accompanying diagram shows the cross section of a long, cylindrical, co-axial cable. The inner conductor has radius R_{I} and the outer conductor exists between radii $R_{\mathrm{O}1}$ and $R_{\mathrm{O}2}$. The magnetic properties of the insulating material separating the two conductors can be ignored. A current I flows in the inner conductor and the same current (in the opposite direction) flows in the outer conductor. The currents are uniformly distributed within each conductor.

Determine the magnetic field for all radial distances r.



Problem 2 (20 points)

a) Refer to the accompanying diagram for light propagating between two media with refractive indices n_1, n_2. What is the optical path length (OPL) for a beam of light which travels between the *fixed* points A and B in terms of the refractive indices, the constants h_1, h_2, d, and the variable x?

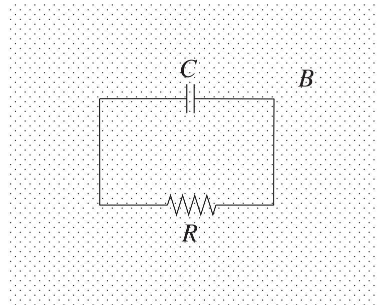


b) Use the OPL above to derive Snell's law using Fermat's principle.

c) Discuss the behavior of a plane wave of light travelling through glass (refractive index 1.6) that encounters a flat glass-water surface (refractive index of water $\sim$ 1.3) at an incident angle of 62 degrees to the surfaces normal.

Problem 3 (20 points)

An RC circuit of area A lies in a plane perpendicular to a uniform magnetic field $\vec{B}$. After the fields magnitude B has been constant for a long time, at $t = 0$ it begins to decrease exponentially, as $B(t) = B_0 \exp(-\kappa t)$. Find the charge on the capacitor, $Q(t)$, for $t \geq 0$. (Define $\gamma = \dfrac{1}{RC}$.) What is the form of $Q(t)$ for short times? What is $Q(t)$ when $\kappa = \gamma$?



Problem 4 (10 points)

Consider an electric dipole with dipole moment $\vec{p}$ formed by two charges $-q$ and $+q$ located a distance $|\vec{a}|$ apart. Let the dipole be placed in a spatially varying electric field $\vec{E}(\vec{r})$. Find the force on the dipole in terms of the dipole moment $\vec{p} = q\vec{a}$. What is the force if $\vec{E}$ is independent of $\vec{r}$?

Problem 5 (30 points)

Consider a small metallic sphere in a uniform field $\vec{E}_0$. Find the electric field $\vec{E}$ inside the sphere. Let the dielectric functions inside and outside the sphere be $\varepsilon_{\text{in}} = \left(1 - \dfrac{\omega_{\text{p}}^2}{\omega^2}\right)\varepsilon_0$ and $\varepsilon_{\text{out}} = \varepsilon_0$. Find the condition on ω so that $|\vec{E}| \to \infty$. The plasma frequency ω_{p} is given by $\omega_{\text{p}}^2 = \dfrac{ne^2}{\varepsilon_0 m}$, where the electrons in the metal have number density n, mass m, and charge $-e$. Make an estimate, for typical metallic densities n, of the value of ω for which $|\vec{E}| \to \infty$. (This problem uses SI units; if you prefer CGS-Gaussian units, then $\varepsilon_{\text{in}} = 1 - \dfrac{\omega_{\text{p}}^2}{\omega^2}$, $\varepsilon_{\text{out}} = 1$, and $\omega_{\text{p}}^2 = \dfrac{4\pi ne^2}{m}$ may be used.)