Department of Physics

Preliminary Exam January 4–8, 2010

Day 3: Quantum Mechanics and Modern Physics

Thursday, January 7, 2010

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Problem 1

(A) Consider a one-dimensional harmonic oscillator with frequency ω that is initially in a state $\mid \psi >$ represented by

$$\mid \psi > = 2 \mid 0 > + \mid 1 >$$

where $\mid 0 >$, $\mid 1 >$ represent the ground state and the first excited state in the energy basis.

(a) What are the possible outcomes of energy measurements in this state, and their respective probabilities ?

(b) Calculate the mean value of energy for this state.

(B) The wavefunction of a particle in one dimension is given by

$$\psi(x) = Ae^{-2x^2}$$

(a) What is the value of A for $\psi(x)$ to be normalized ?

(b) Calculate the expectation values of p^2 for this state.

Useful integral: $\displaystyle\int_0^{\infty} e^{-bx^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{b}}$

Problem 2

In quantum mechanics, the x, y, and z components of the spin angular momentum of a non-relativistic spin $\frac{1}{2}$ particle (say, an electron) can be represented by

$$\vec{S} = \frac{1}{2}\hbar\vec{\sigma}$$

where $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(a) Show that S_x, S_y, and S_z satisfy the commutation relations of the angular momentum.

(b) Find the eigenvalues of the operator S_y.

(c) The states of the electron can be represented by the 2-dimensional column vectors. Find the eigenstates of S_y.

(d) Suppose the electron is in the spin state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$. If S_y is measured, what is the probability of getting the result $\dfrac{\hbar}{2}$?

Problem 3

The accompanying figure depicts the incoming direction for a plane wave beam of quantum mechanical particles traveling in the x-y plane as shown. The potential energy is given by

$$V = \begin{cases} 0, & x < 0, \\ -V_0, & x > 0. \end{cases}$$

The left-hand side of the figure corresponds to $x < 0$. The incident particles have energy $E > 0$, and with V_0 constant and positive. The incident component of the wave function can be assumed to be expressed in the form

$$\Psi_I(x, y) = A\, e^{i(k_x x + k_y y)}.$$

1. Write down the appropriate boundary conditions satisfied by the incident, reflected, and transmitted components of the total wave function.
2. Calculate the transmission coefficient, T, and the reflection coefficient, R, defined in terms of the ratios of the appropriate probability fluxes. The probability flux is the probability per unit time that a particle will cross some reference location traveling in a particular direction.
3. Show that $T + R = 1$.

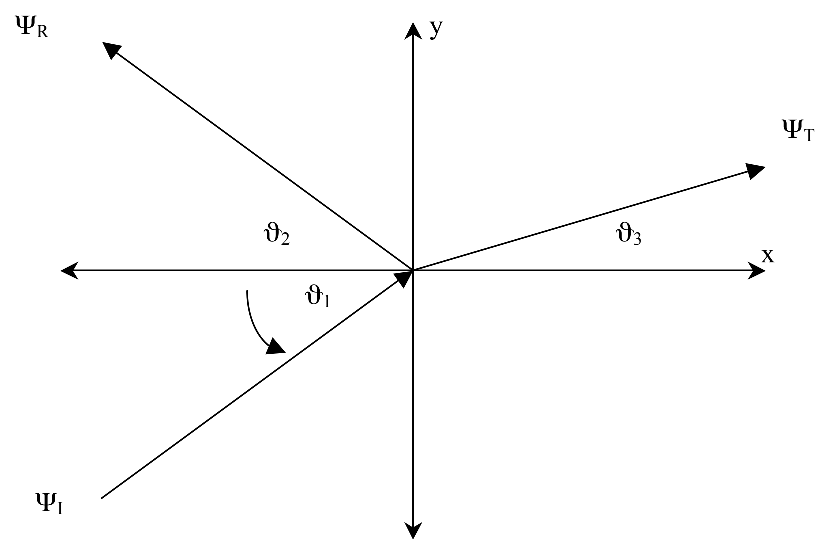

Figure 1: Illustration of incident, reflected, and transmitted components of particle wave function at potential boundary.

Problem 4

In a reactor and separation-plant complex, nuclide A is produced in pure form, and decays into nuclide B by beta-decay with a half-life of 23 minutes. Nuclide B decays with a half-life of 23 days by beta- and gamma-ray emission to a stable nuclide C. A radioactive sample containing the nuclides A, B, and C produced at the plant was shipped to a physics professor some distance away.

1. How many nuclei of each type (A, B, and C) are present at a time T after initial production of the pure nuclide A, assuming the initial sample had N_0 atoms of nuclide A?

2. The documents that came with the sample stated that the gamma-ray emission of the sample taken 11.5 minutes after production of the initially pure batch of nuclide A was 1000 counts/sec. When the physics professor received the sample, he measured the gamma-ray emission rate and found it was also 1000 counts/sec. How much time had passed since the initial pure sample had been produced?

   (HINT: the time is very long compared to the half-life of nuclide A, and greater than the half-life of nuclide B.)

Problem 5

Compute the recoil kinetic energy of the proton in MeV in neutron β-decay when the neutrino has the maximum energy. M_n=939.57 MeV/c^2, M_e=0.511 MeV/c^2, M_p=938.27 MeV/c^2. Ignore transverse momentum of the proton.