Department of Physics

Preliminary Exam January 4–8, 2010

Day 4: Thermodynamics and Statistical Physics

Friday, January 8, 2010

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Useful information:

Boltzmann constant:	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Ideal gas constant:	$R = 8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$
	$1\,\text{liter} = 10^{-3}\,\mathrm{m^3}$
	$1\,\text{atm} = 1.013 \times 10^5\,\mathrm{Pa}$
Stirling's approximation:	$\ln N! = N \ln N - N$

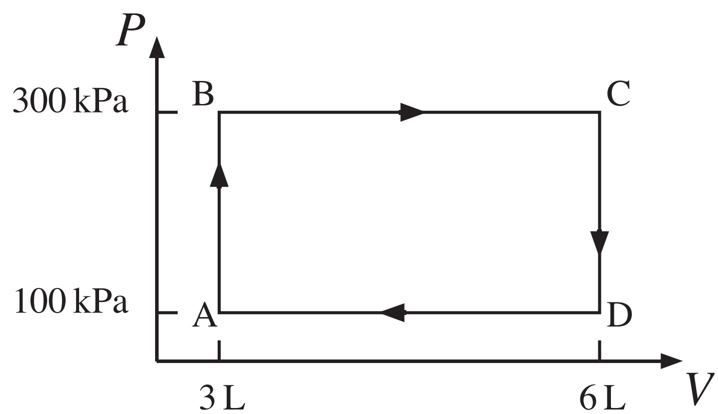
Problem 1

A monoatomic ideal gas is taken clockwise around the rectangular path as shown in the figure.

(a) How much work is done by the gas per cycle?

(b) If there is 1.0 mole of gas, what is the maximum temperature reached?

(c) What is the total heat input to the ideal gas from "A $\rightarrow$ B $\rightarrow$ C"?



Problem 2

In the course of pumping up a bicycle tire, one liter of air at atmospheric pressure is compressed adiabatically to a pressure of 7 atm. (Air is mostly diatomic nitrogen and oxygen.)

(a) What is the final volume of this air after compression?

(b) How much work is done in compressing the air?

(c) If the temperature of the air is initially 300 K, what is the temperature after compression?

Problem 3

A system consists of N weakly interacting, distinguishable particles, each of which can be in either of two states with respective energies ε_1 and ε_2, where $\varepsilon_1 < \varepsilon_2$.

(a) Without explicit calculation, make a qualitative plot of the mean energy $\overline{E}$ of the system as a function of its temperature T. What is $\overline{E}$ in the limit of very low and very high temperatures?

(b) Using the result of (a), make a qualitative plot of the heat capacity C_V (at constant volume) as a function of the temperature T.

(c) Calculate explicitly the mean energy $\overline{E}(T)$ of this system.

(d) Verify that your expressions exhibit the qualitative features discussed in (a).

Problem 4

Two identical copper blocks of mass $m = 1.5\,\mathrm{kg}$ are in a thermally insulated box, separated by an insulating shutter. One of the blocks is at 60°C and the other at 20°C. The specific heat of copper is $386\,\mathrm{J\,kg^{-1}K^{-1}}$. (In each part, show explicitly how you arrived at the answer.)

(a) When we lift the shutter, the blocks eventually come to an equilibrium temperature T_f. What is this equilibrium temperature T_f?

(b) What are the entropy changes of each block and the entropy change in the two-block system during this process?

(c) What is the implication of the entropy change in the two-block system that you just calculated in (b) in terms of reversibility or irreversibility of the process?

(d) What is the maximum amount of work that could have been extracted from this system if, instead of placing the two blocks in contact with each other, they were used as the energy source for an ideal engine?

Problem 5

Consider a monoatomic ideal gas of N atoms of mass m confined in a volume $V = L^3$ at absolute temperature T. The energy of the molecule is

$$\varepsilon = \frac{\vec{p}^2}{2m} = \frac{\hbar^2 \vec{\kappa}^2}{2m},$$

where $\vec{p}$ denotes its (three component) momentum vector and $\vec{\kappa}$ the wave vector. The solution leads to quantum states with

$$\kappa_x = \frac{\pi}{L} n_x, \quad \kappa_y = \frac{\pi}{L} n_y, \quad \kappa_z = \frac{\pi}{L} n_z,$$

where n_x, n_y and n_z are 1, 2, 3, $\ldots$. Using the approximations

$$\sum_{n_x=1}^{\infty} \mathrm{e}^{-\frac{\beta \pi^2 \hbar^2}{2mL^2} n_x^2} \cong \frac{L}{h} \left(\frac{2\pi m}{\beta} \right)^{1/2},$$

it is possible to show that the partition function for a single molecule ζ in the classical approximation is $\zeta = \dfrac{V}{h^3}(2\pi m k T)^{3/2}$.

(a) Obtain an expression for the chemical potential μ of this gas. You may use the classical approximation for the partition function taking into account that the particles are indistinguishable.

(b) A gas of N' such weakly interacting particles, adsorbed on a surface of area A on which they are free to move, can form a two-dimensional ideal gas on such a surface. The energy of an adsorbed molecule is then $\dfrac{\vec{p}^2}{2m} - \varepsilon_0$ where $\vec{p}$ denotes its (two component) momentum vector and ε_0 is the binding energy which holds the molecule on the surface. Find the partition function and the chemical potential μ' of this adsorbed ideal gas.

(c) At the temperature T, the equilibrium condition between molecules adsorbed on the surface and the molecules in the surrounding three-dimensional gas can be expressed in terms of the respective chemical potentials. What is this equilibrium condition? Use this condition to find at temperature T the mean number of molecules adsorbed per unit area (N'/A) when the mean pressure in the surrounding gas is P.