

Department of Physics

Preliminary Exam January 3–7, 2011

Day 1: Classical Mechanics

Monday, January 3, 2011

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

There are five problems. Answer all five. Each problem carries 20 points. Include all relevant intermediate steps.

Problem 1

Friction. A block of mass m rests on a horizontal board, 5 m long, that is itself lying on a concrete surface. One end of the board is gradually lifted until the block starts to slide downhill. Let the coefficients of static and kinetic friction be identical, and symbolized by the Greek letter μ . Call the angle between the board and ground θ , and represent the acceleration due to gravity by g .

- (a) Calculate the angle (in degrees) at which the block just begins to slide when $g = 9.8 \text{ m/s}^2$ and $\mu = 0.35$.
- (b) Now let the board be propped up at one end so that $\theta = 42^\circ$. The mass is released from rest at the high end of the board and slides downhill. Calculate the speed of the mass after it has slid a distance $d = 1.2 \text{ m}$ from its starting position.
- (c) Repeat part (a) for an initially horizontal board on the planet Mars, where $g = 3.7 \text{ m/s}^2$. Use the same value of μ as before.

Problem 2

Circular orbits. When a small moon of mass m orbits a massive planet of mass M , the center of mass of the pair is essentially at the exact center of the planet. Assume that the two bodies are spherical and that the moon follows a circular path around the planet. Denote the distance between their centers by d , and the universal gravitational constant by G (not the acceleration g).

- (a) Derive an expression for the orbital period of the moon (T) in terms of (some of) the above physical quantities. Mathematical constants and numbers may also be present in the expression.
 - (b) Now relax the previous assumption that $M \gg m$, allowing for the possibility that the masses are comparable in size. As one consequence, the two masses will follow circular paths about the center of mass (CM) of the pair. Let r stand for the small mass's distance from the CM, and R for the large mass's distance from the CM. Thus, $d = r + R$. Find expressions for r and R in terms of d , m and M .
 - (c) Derive a new, improved expression for T in which both masses appear.
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Problem 3

Harmonic motion in the plane. The oscillation frequency of an isotropic two-dimensional harmonic oscillator is 20 Hz. The initial conditions are given as follows:

$$x_0 = 0.2 \text{ m}, \quad \dot{x}_0 = 0; \quad y_0 = 0, \quad \dot{y}_0 = 20 \text{ m/s}.$$

Find the equation of the trajectory and make a plot of the trajectory.

Problem 4

Rigid-body rotation. Consider a thin, uniform plate, of rectangular shape with dimensions a and b . The plate lies in the first quadrant of the x - y plane, with one corner at the origin and the plate edges running along the coordinate axes. The plate has negligible thickness and a total mass m (constant density). Now, let this plate be caused to rotate about a body diagonal passing through the coordinate origin at a constant angular speed ω . (The coordinate system is fixed relative to the body.)

- (a) Make a sketch to depict the geometry of the problem as described above.
- (b) Compute the coordinates of the center of mass.
- (c) With respect to the center of mass, determine, by any method, the principal axes.
- (d) Determine the angular momentum vector (magnitude and direction), relative to the coordinate-system origin.
- (e) Compute the kinetic energy of this rotating plate.
- (f) Finally, compute the magnitude of the torque which must be exerted on the plate in this situation to maintain the constancy of ω along the axis prescribed above.

Problem 5

Formal results. If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange's equations, show by direct substitution that

$$L' = L + \frac{dF(q_1, \dots, q_n, t)}{dt}$$

also satisfies Lagrange's equations where F is any arbitrary, but differentiable, function of its arguments.