Department of Physics

Preliminary Exam January 3–7, 2011

Day 2: Electromagnetism and Optics

Tuesday, January 4, 2011

9:00 a.m. – 12:00 p.m.

Instructions:

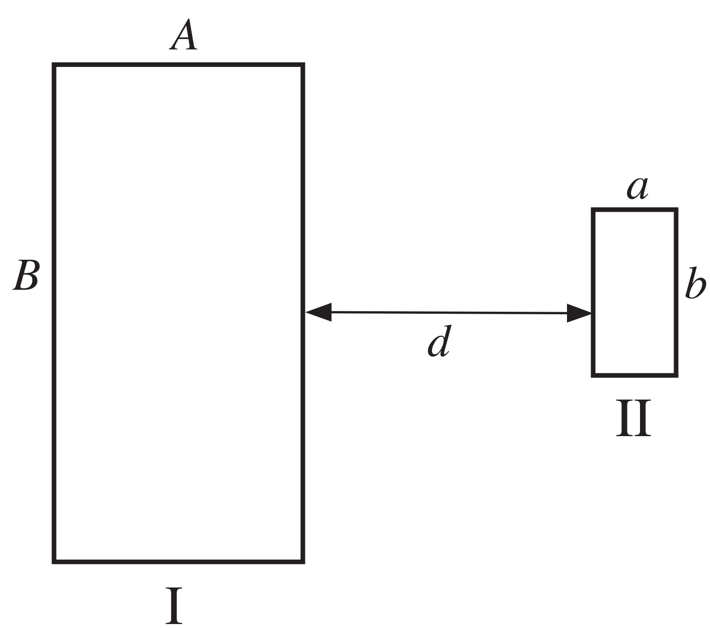
1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Problem 1

The accompanying diagram shows two rectangular circuits of wire (labeled I and II) which are coplanar. Circuit I has length B and width A, circuit II length b and width a. The circuits are separated by a distance d as shown.

(a) Find the mutual inductance between the two circuits in the limit that $B \to \infty$.

(b) Determine the magnitude of the current in circuit I if it has a resistance of R and the current in circuit II is changing at a rate of λ.



Problem 2

A thin spherical shell of radius a is divided into two hemispheres which are insulated from one another. One hemisphere is kept at a potential V and the other is grounded.

(a) Using a separation of variables technique find the monopole, dipole and quadrupole terms for the potential outside of the sphere.

(b) Determine the total charge on the shell and the dipole moment of the charge.

Potentially useful information:

The solution of Laplace's equation for an axially symmetric potential is:

$$\phi(r,\theta) = \sum_{l=0}^{\infty}(\alpha_l r^l + \beta_l r^{-(l+1)})P_l(\cos\theta),$$

where α_l and β_l are constants.

The first three Legendre polynomials are:

$$P_0(\cos\theta) = 1, \quad P_1(\cos\theta) = \cos\theta, \qquad P_1(\cos\theta) = \tfrac{1}{2}(3\cos^2\theta - 1).$$

The Legendre polynomials have the orthogonality property:

$$\int_0^{\pi} P_l(\cos\theta)P_m(\cos\theta)\sin\theta\,\mathrm{d}\theta = \frac{2}{2l+1}\delta_{lm}.$$

Problem 3

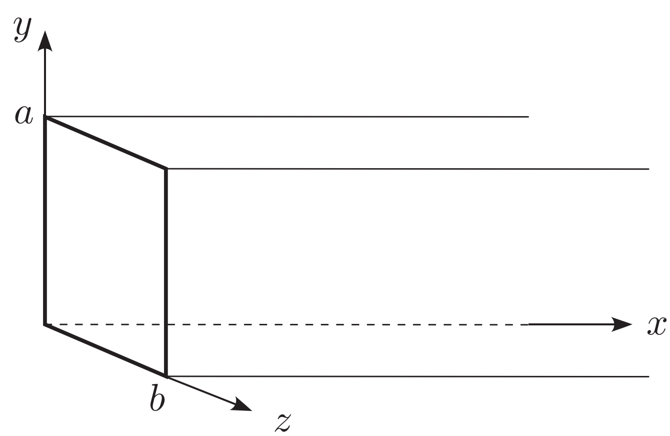
A thin circular ring of radius R lies in the x-y plane, centered on the origin. The ring has a uniformly distributed total charge Q, and is rotating about the z axis at angular speed ω. Find the electric and magnetic fields, produced by the rotating ring of charge, at an arbitrary point on the z axis.

Problem 4

The transmission of a plane wave through a Fabry–Pérot interferometer, consisting of two parallel, partially transmitting, planar mirrors separated by a distance L, is measured as the frequency of the normally incident light is varied. If the mirrors have reflectivities $R_1 = 97\%$ and $R_2 = 99\%$, and no absorption or scattering losses (each photon incident on a mirror is either transmitted or reflected, i.e., $T_i + R_i = 1$ for $i = 1$ or 2), what is the maximum transmission of the interferometer?

Problem 5

Work out the theory of Transverse Magnetic (TM) modes ($B_x = 0$) for a rectangular waveguide (see figure below).



In particular, solve the differential equation for E_x,

$$\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \left(\frac{\omega}{c}\right)^2 - \kappa^2\right] E_x = 0,$$

subject to the boundary conditions, $E_x = 0$ at $y = 0$, $y = a$, $z = 0$, and $z = b$. Determine the functional form for E_x, the cutoff frequencies, and the wave and group velocities. What is the lowest TM mode?