Department of Physics

Preliminary Exam January 3–7, 2011

Day 4: Thermodynamics and Statistical Physics

Friday, January 7, 2011

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Useful data:

Boltzmann's constant . . . . . $k = 1.38\times10^{-23}\,\mathrm{J/K}$
Gas constant . . . . $R = 8.31\,\mathrm{J/mol\cdot K}$
Unified mass unit . . . . $u = 1.66\times10^{-27}\mathrm{kg}$

Gaussian integrals:

$$\int_0^\infty \exp(-\lambda x^2)\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$$
$$\int_0^\infty x\exp(-\lambda x^2)\mathrm{d}x = \frac{1}{2\lambda}$$
$$\int_0^\infty x^2\exp(-\lambda x^2)\mathrm{d}x = \frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}}$$

Problem 1 (15 points)

A system consisting of 2 kg of water at 20°C is brought in contact with a heat reservoir at 50°C. The temperature of the system increases until it reaches the temperature of the reservoir. Consider the specific heat capacity as a constant equal to $1\,\mathrm{cal\,g^{-1}K^{-1}}$ ($4.18\times10^3\mathrm{J\,kg^{-1}K^{-1}}$ in SI units).

(a) Calculate the heat transferred from the heat reservoir to the system.

(b) Calculate the entropy change in the heat reservoir.

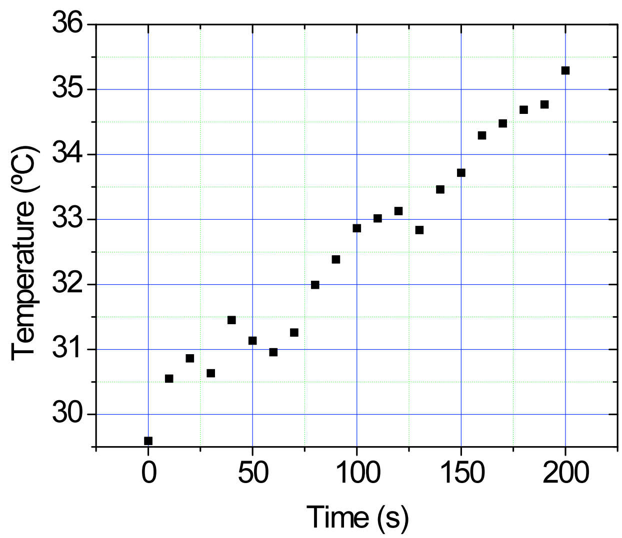
(c) Calculate the entropy change in the system.

(d) What is the entropy change in the universe?

(e) Is this process reversible or irreversible? Justify.

Problem 2 (20 points)

An experiment is performed with 2 moles of an ideal gas in an adiabatic container of $0.5\,\mathrm{m}^3$ provided with a piston, initially locked in place. A heater is introduced in the container and the current is switched on at the instant $t = 0\,\mathrm{s}$, dissipating a power of $1\,\mathrm{W}$. A thermocouple registers the following temperature profile as a function of the time since the heater was switched on:



(a) Based on the data presented in the figure above, obtain an estimate for the heat capacity and for the specific heat capacity at constant volume for this gas.

(b) If the experiment is repeated with the same initial conditions, but with the piston unlocked, *i.e.*, able to move in such a way that the pressure is constant, would the temperature be higher or lower than in the previous case at $t = 200\,\mathrm{s}$? Justify.

(c) Calculate the final temperature of the gas in the situation described in part (b).

(d) Is this most likely a monoatomic or diatomic gas? Justify your answer.

Problem 3 (20 points)

Please answer the following questions on the properties of a Carnot cycle and the efficiency of a Carnot engine.

(a) What thermodynamic processes are involved in a Carnot cycle?

(b) Illustrate the Carnot cycle on a P–V diagram and an S–T diagram. Please label the pairs of corresponding corners with the letters A to D.

(c) In which processes is work put in or extracted?

(d) Derive the efficiency of an engine using the Carnot cycle. Express the efficiency as a function of temperatures.

Problem 4 (20 points)

Consider a box of volume V containing N gas molecules of mass m, at temperature T.

(a) Derive the normalized Maxwell distribution of the molecular velocity, $P(v_x, v_y, v_z)$. Integration of P should equal to N. You may assume that the velocity components are independent, and that the distribution depends only on speed or kinetic energy.
You may need to use Boltzmann's distribution for classical particles.

(b) When a particular solid surface is exposed to this gas, it begins to absorb molecules at a rate W (molecules $\cdot\, \mathrm{s}^{-1}\mathrm{m}^{-2}$). A molecule has absorption probability of 0 for a normal velocity component less than a threshold v_T, and absorption probability 1 for a normal velocity greater than v_T. Derive an expression for W simplifying it as far as you can.

(c) Derive Maxwell's distribution of molecular speed, $n(v)$. You may start from the Maxwell distribution of velocity in part (a).

(d) Derive the most probable speed from Maxwell's distribution of molecular speed.

(e) Calculate the most probable speed for H_2 at room temperature.

Problem 5 (15 points)

Consider a system consisting of two particles, each of which can be in any of one of three quantum states of respective energies 0, ε, and 3ε. The system is in contact with a heat reservoir at temperature $T = (k\beta)^{-1}$.

(a) Write an expression for the partition function Z if the particles obey classical Boltzmann statistics and are considered distinguishable.

(b) What is Z if the particles obey Bose-Einstein statistics?

(c) What is Z if the particles obey Fermi-Dirac statistics?

Problem 6 (10 points)

For Fermi-Dirac distribution functions shown in the following figure, order from small to large (a) the Fermi temperatures and (b) the temperatures of the three systems. Please justify your solution.