Department of Physics

Preliminary Exam January 3–7, 2012

Day 1: Classical Mechanics

Tuesday, January 3, 2012

9:00 a.m. – 12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Problem 1

Boulder orbit (15 points)

Consider a satellite of mass m on a low, circular orbit around Earth (mass M). Naturally, $m << M$. "Low" means that the satellite's height (h) above the ground is much smaller than Earth's radius (R). Let T be the orbital period.

Now consider a roughly spherical boulder in outer space, far from any planet or star. Let there be a pebble following a low, circular orbit around this boulder.

If the satellite and pebble have equal periods, what must be true about the Earth and the boulder? Derive this result analytically.

Problem 2

Gravitational Attraction of a Sphere (25 points)

Consider a point mass m and a sphere of radius a and mass M whose internal density distribution $\rho(r)$ is spherically symmetric (depending only on r, the distance from the sphere's center). Let m be outside the sphere, at a distance R from the sphere's center. Show that the gravitational pull of the sphere on m is equivalent to that of a point mass of mass M at the sphere's center. Clearly label your work.

Hints:

a) Think of the sphere as a set of concentric, spherical shells.

b) Law of cosines for triangle with sides a, b, c and angle C opposite side c: $c^2 = a^2 + b^2 - 2ab\cos C$

c) Derive an expression for the potential energy of attraction of one spherical shell and the mass m

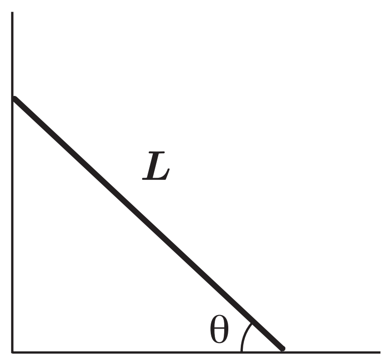
Problem 3

(20 points)

A uniform ladder of mass M and length L is placed with one end against a frictionless wall and the other end on a frictionless floor. The ladder initially makes an angle θ_0 with the floor, as shown below. The ladder is released and slides under the influence of gravity.

a) Write the Lagrangian for the sliding ladder as a function of θ.

b) At what angle θ does the ladder lose contact with the wall? (the moment of inertia of a uniform rod with mass M and length L rotating about an axis through its center of mass is $I = ML^2/12$).



Problem 4

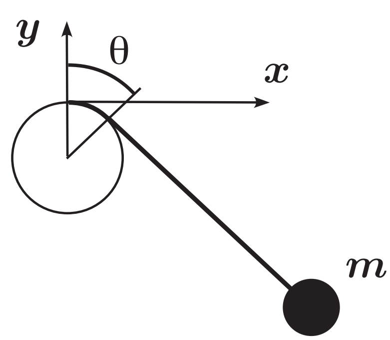
(25 points)

Consider a pendulum built from a mass m attached to one end of a massless, unstretchable string, whose other end is attached to the uppermost point of a fixed, vertical disk of radius R, as shown in the figure below. The initial position of the system is shown in the figure. Assume that the total length of the string is l, and that $\pi R < l$.

a) Find the equation of motion in terms of angle θ as shown in the figure.

b) What is the equilibrium angle θ_0?

c) Find the frequency of oscillations about the equilibrium angle.



Problem 5

Small Oscillation (15 points)

Three particles of equal mass m are constrained to lie on a circle. They are connected by identical springs of spring constant k on the circle. The radius of the circle is a.

a) Construct the Lagrangian for the system.

b) Find the eigenfrequencies of the system.

c) Find the general solution for the motion of these particles.