# Department of Physics

# Preliminary Exam January 3–7, 2011

## Day 3: Quantum Mechanics and Modern Physics

## Thursday, January 6, 2011

## 9:00 a.m. – 12:00 p.m.

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**Instructions:**

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

*Answer Problems 1-4 and either Problem 5 or Problem 6.*
*Each problem is worth 20 points for a total of 100 points.*
*Only one solution from Problems 5 and 6 will be accepted.*

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**Problem 1**

Secondary cosmic ray muons created in the Earth's atmosphere typically have relativistic energies and are the dominant type of particle formed in cosmic ray air showers to reach the Earth's surface. Muons have a rest mass of 106 MeV/c$^2$ and an average lifetime of $2.2 \times 10^{-6}$ s. Determine the range in the Earth's atmosphere of a muon with a kinetic energy of 500 MeV, as measured by a stationary observer on Earth. (Neglect collisional effects of the muon traveling through the atmosphere, i.e., assume the atmosphere is a vacuum.)

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**Problem 2**

(a) Determine the form of the wave function as $x \rightarrow \pm\infty$ corresponding to a single particle bound state in a potential $V(x)$ such that $V(x = \pm\infty) = 0$.

(b) Using (a), prove that bound states of a one-dimensional Hamiltonian are nondegenerate.

(c) Given the two lowest bound state wave functions for a particle moving in a one-dimensional harmonic oscillator potential ($V(x) = \frac{1}{2} C\, x^2$) below, show that the asymptotic condition of part (a) and the nondegeneracy property of part (b) still hold.

$$\Psi_0 = A_0\, e^{-\frac{1}{2}u^2}, \quad \Psi_1 = A_1\, u\, e^{-\frac{1}{2}u^2}, \quad \text{where } u = \left[(Cm)^{1/4}/\hbar^{1/2}\right] x.$$

**Problem 3**

A particle of mass $m$ moves freely in a two-dimensional square box with sides $L$.

(b) Determine the general expressions for the wave functions and energies as functions of quantum numbers $n_1$ and $n_2$ (assume the box is in the $xy$-plane with $0 \leq x \leq L$ and $0 \leq y \leq L$).

(b) Explain if either (or both) of the ground state and first excited state energies are degenerate.

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**Problem 4**

A spin–1/2 particle is described by the Hamiltonian

$$H = a\left(\sqrt{3}S_x + S_z\right)$$

where $a$ is a constant and ($S_x$, $S_y$, $S_z$) are the three components of the spin operator:

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Determine the eigenvalues and eigenvectors of $H$.

(b) What is the probability of measuring the spin to be $+\hbar/2$ along the $z$–direction for these two eigenstates?

(c) One linear combination of $S_x$, $S_y$ and $S_z$ is a conserved quantity in this problem. Identify this combination. What additional spin observable is conserved here?

## Problem 5

(a) Create a table listing the four fundamental forces of nature in order of decreasing strength.

(b) Assume that the coupling constant for the strongest of the forces is equal to 1 and provide the relative values of the coupling constants for each of the other forces at ordinary energies.

(c) For each force, provide typical values of the range over which the force acts.

(d) For each force, list typical lifetimes or durations of the force's interaction.

(e) Where known, name the force carrier particle (gauge boson) that mediates that force's interaction.

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## Problem 6

The figure below shows the mass attenuation coefficients for energetic photons ($x$-rays and $\gamma$-rays) in aluminum and lead. The figure is divided into three regions. Describe the physical process that occurs in each of the three regions. Explain the sharp peaks seen in Region I. **Hint:** consider the three processes by which energetic photons ($x$-rays and $\gamma$-rays) interact with matter.