Department of Physics

Preliminary Exam January 2–5, 2013

Day 1: Classical Mechanics

Wednesday, January 2, 2013

9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *five* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Problem 1

(a) For a particle in an attractive potential $-k/r$ (Kepler problem), the trajectory is given by

$$\frac{C}{r} = 1 + \epsilon \cos\theta, \qquad C, \epsilon \text{ constants.}$$

Here, r denotes the radial distance. Find the condition on ϵ so that the trajectory is a hyperbola.

(b) Show that the Lorentz transformations

$$x_1' = \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}}, \quad x_2' = x_2, \quad x_3' = x_3, \quad t' = \frac{t - \frac{vx_1}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

can be written as

$$\begin{aligned} x_1' &= x_1 \cosh\alpha - ct \sinh\alpha, \\ x_2' &= x_2, \\ x_3' &= x_3, \\ t' &= t \cosh\alpha - \frac{x_1}{c} \sinh\alpha. \end{aligned}$$

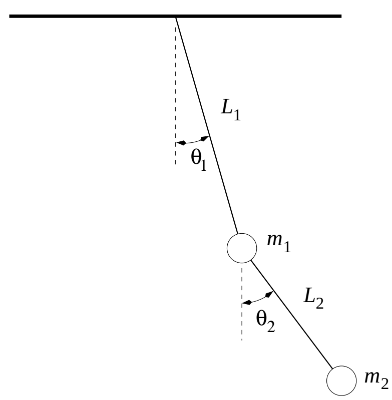
Find α. Show that these transformations correspond to a rotation through an angle, $i\alpha$, in four-dimensional space.

Problem 2

(a) A non-relativistic particle of mass m collides with another particle of mass $M = am$, initially at rest. The collision is elastic and central. Determine how the energy, lost by the moving particle, depends on the mass ratio a, and find the value of a for which the energy loss is largest.

(b) Cosmic-ray photons from space are bombarding your laboratory and smashing massive objects to pieces. Your detector indicates that two fragments, each of mass m_0, depart such a collision at speed $0.6c$ at angles 60° relative to the photon's original direction of motion. In terms of m_0 and c, what is the energy of the cosmic-ray photon and what is the mass M of the particle being struck by the photon?

Problem 3



Consider the case of a double pendulum shown where the top pendulum has length L_1 and the bottom length is L_2, and similarly the bob masses are m_1 and m_2. The motion is only in the plane. Assume small oscillations.

(a) Find the frequencies of the normal modes.

(b) Describe the motion when $m_2 = 0$.

(c) Describe the motion when $m_2 = m_1 = m$.

Problem 4

(a) Show that the relativistic Lagrangian

$$L = -mc^2\sqrt{1-\frac{v^2}{c^2}} - V(\mathbf{r}), \quad \mathbf{v} = \frac{d\mathbf{r}}{dt}$$

yields the correct equation for the relativistic momentum

$$\frac{d}{dt}\left(\frac{m\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = -\nabla V(\mathbf{r}).$$

(b) Find the Hamiltonian associated with the Lagrangian

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 - \Phi(\mathbf{r},t) + \mathbf{A}(\mathbf{r},t)\cdot\dot{\mathbf{r}},$$

where Φ and $\mathbf{A}$ are some given functions of $\mathbf{r}$ and t.

Problem 5

A square homogeneous slab of thickness a and length $4a$ is placed atop a fixed cylinder of radius R whose axis is horizontal.

(a) Show that the condition for stable equilibrium of the slab, assuming no slipping, is $R > a/2$.

(b) What is the frequency of small oscillations?

(c) Sketch the potential energy U as a function of the angular displacement θ for $R > a/2$.