Department of Physics

Preliminary Exam January 2–5, 2013

Day 2: Electricity, Magnetism and Optics

Thursday, January 3, 2013

9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Problem 1

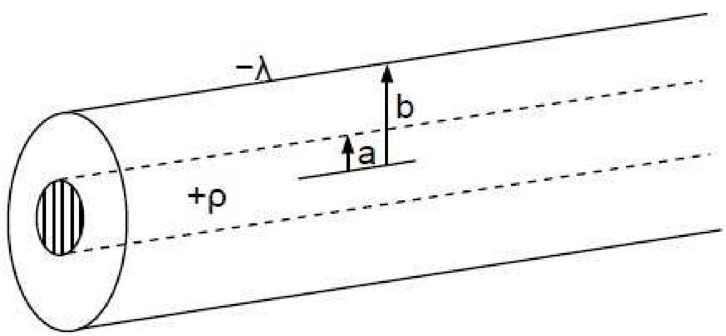
A long coaxial cable (see figure) carries a uniform positive *volume* charge density ρ on the inner cylinder (radius a), and a uniform negative *surface* charge density, $-\lambda$, on the outer cylindrical shell (radius b). Find the electric field vector in each of the following regions:

(a) Inside the inner cylinder ($r < a$)

(b) Between the cylinders ($a < r < b$)

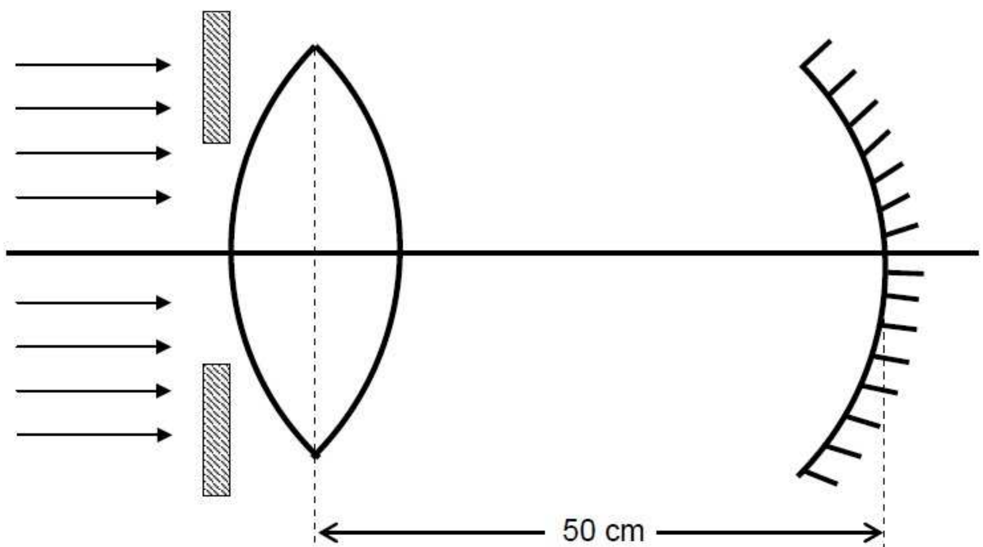
(c) Outside the cable ($r > b$).

(d) Determine the relationship between ρ and λ that will make the cable electrically neutral for $r > b$.



Problem 2

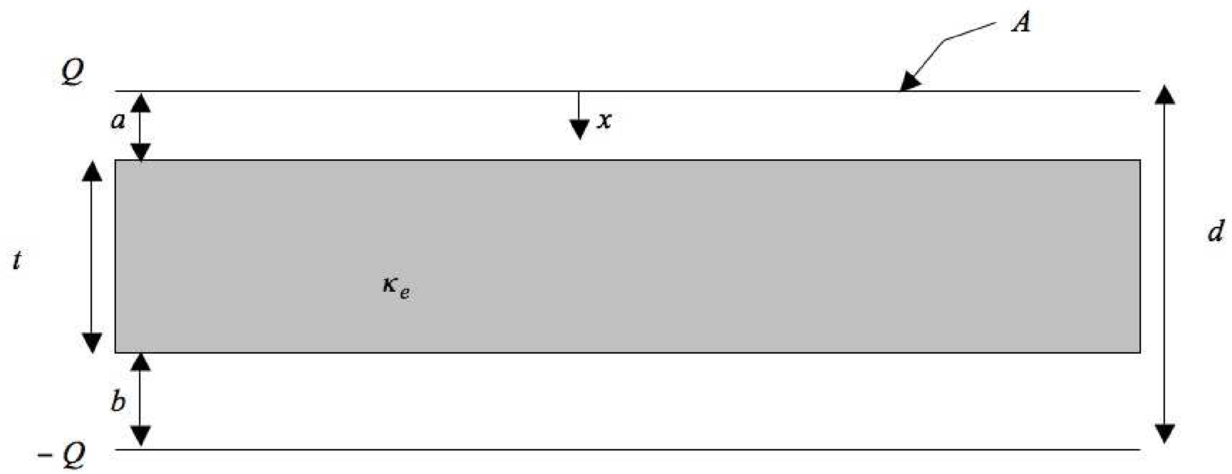
(a) A lens of focal length +20 cm is placed 50 cm to the left of a concave mirror whose radius of curvature is −20 cm (see figure). Light rays parallel to the optic axis are incident on the lens from the left. Calculate the positions of the points on the optic axis at which the light is focused by this lens-mirror system. Assume the lens and mirror are both optically thin and the aperture only allows paraxial rays to enter the lens. Use a ray diagram to illustrate your answer.



(b) Considering only the fact that the light consists of red and blue wavelengths, describe how many focused points will now appear on the optic axis for this thick-lens/thin-mirror system.

Problem 3

A slab of dielectric of thickness t is inserted into a parallel plate capacitor of plate separation d and plate area A as shown in the figure below. The surfaces of the slab are parallel to the plate surfaces. Find **D**, **E**, and **P** as functions of x. (Express them in terms of Q.) Find the capacitance of this system. Verify that your results for C reduces to the proper values when $t = d$. In this problem, κ_e is the dielectric-constant ratio ϵ/ϵ_0.



Problem 4

"Eddy currents" are currents produced in conductors as a result of Faraday's Law.

(a) Write Faraday's Law in differential form.

(b) A very thin conducting disc of radius a and conductivity σ lies in the x-y plane with the origin at its center. A spatially uniform induction is also present and given by $\mathbf{B} = B_0 \cos(\omega t + \alpha)\hat{\mathbf{z}}$. Find the induced current density $\mathbf{J}_f$ produced in the disc.

Problem 5

For waves propagating along the z-axis in a waveguide of uniform cross section, the wave equation for the full electromagnetic field simplifies to the following wave equation for $\psi = E_z$ (B_z) for TM (TE) waves:

$$\left(\nabla_t^2 + \gamma^2\right)\psi = 0$$

where

$$\gamma^2 = \mu\epsilon\frac{\omega^2}{c^2} - k^2 \qquad \textit{and} \qquad \nabla_t^2 = \nabla^2 - \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2}$$

in cylindrical coordinates.
Here, μ is the permeability, ϵ the permittivity, c the speed of light, ω the angular frequency of the wave and k the wave-vector along the z-axis. Likewise, the boundary conditions are

$$\psi|_{\text{surface}} = 0 \quad \left(\left.\frac{\partial\psi}{\partial n}\right|_{\text{surface}} = 0\right) \qquad \text{for TM (TE) waves.}$$

If the waveguide is a right circular cylinder of radius L, show that the radial solution obeys the Bessel equation

$$\frac{\partial^2 J}{\partial x^2} + \frac{1}{x}\frac{\partial J}{\partial x} + \left(1 - \frac{\nu^2}{x^2}\right)J = 0.$$

Find expressions for the allowed frequencies and, in particular, the infinite-wavelength cutoff frequencies of the various TE and TM modes.

Problem 6

Two concentric conducting spheres of radius a and b with $a < b$ have between them a non-uniform dielectric of dielectric constant given by $\epsilon = \epsilon(\theta, \phi) = \sum c_{lm} Y_{lm}(\theta, \phi)$. Assume that the field between the spheres is radial. Find an expression for the capacitance in terms of a, b, and the appropriate c_{lm}.