Department of Physics
Preliminary Exam January 2–5, 2013
Day 4: Thermodynamics and Statistical Physics
Saturday, January 5, 2013
9:00 a.m. – 12:00 p.m.

Instructions:

1) Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions. Select the *five* you want graded and hand in a paper *marked with an "X"* for the problem you don't want graded.
2) Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3) The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.
4) If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5) The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6) The use of electronic calculators is permissible and may be needed for some problems. However, obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

*Do **only five (5)** (any five) of the six problems.*
*Hand in a paper **marked with an "X"***
for the problem you don't want graded.
The five graded problems all carry equal weight.

Boltzmann's constant:	$k_\mathrm{B} = 1.38 \times 10^{-23}\,\mathrm{J/K}$
Gas constant:	$R = 8.31\,\mathrm{J/(mol{\cdot}K)}$
Unified mass unit:	$\mathrm{u} = 1.66 \times 10^{-27}\,\mathrm{kg}$

Gaussian integrals:

$$\int_0^\infty \exp(-\lambda x^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}} \qquad \int_0^\infty x^3\exp(-\lambda x^2)\,dx = \frac{1}{2\lambda^2}$$

$$\int_0^\infty x\exp(-\lambda x^2)\,dx = \frac{1}{2\lambda} \qquad \int_0^\infty x^4\exp(-\lambda x^2)\,dx = \frac{3}{8}\sqrt{\frac{\pi}{\lambda^5}}$$

$$\int_0^\infty x^2\exp(-\lambda x^2)\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}} \qquad \int_0^\infty x^5\exp(-\lambda x^2)\,dx = \frac{1}{\lambda^3}$$

Problem 1

Good quality vodka consists of 50% ethyl alcohol and 50% water by volume (bad quality vodka contains additional nasty stuff). How much profit per liter would you make if you purchased the vodka at a cost of \$10.00 per liter at 0°C and sold it at \$10.00 per liter at 25°C? The coefficient of volume expansion of ethyl alcohol is $7.5\times10^{-4}\,{}^\circ\mathrm{C}^{-1}$, while that for water is $2.1\times10^{-4}\,{}^\circ\mathrm{C}^{-1}$.

Problem 2

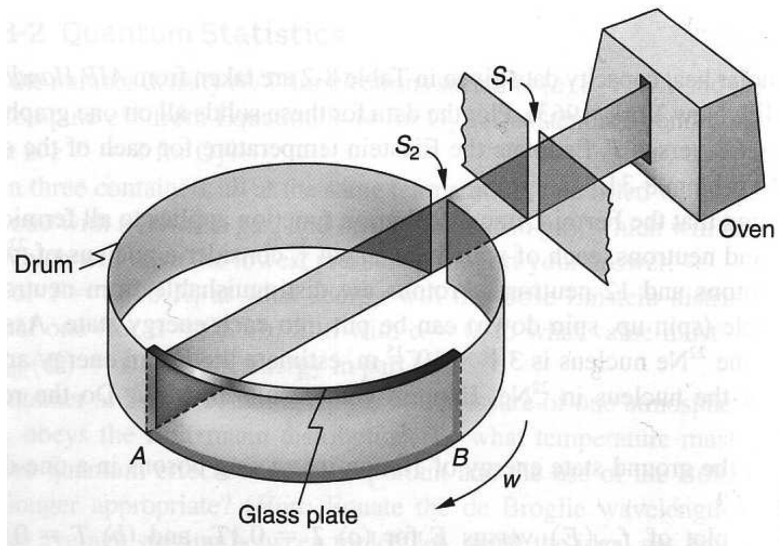
Write down the Maxwell–Boltzmann, Bose–Einstein, and Fermi–Dirac distribution functions. Sketch a plot of these three distributions, $n(E)$ as a function of E/kT from 0 to 5 and a degeneracy constant of $\alpha = -1$, ($\alpha \equiv -\mu/kT$). Name the defining characteristics of the particles/systems described by each type of distribution. Compare the three functions for $E \ll kT$ and for $E \gg kT$.

Problem 3

The device shown in the figure was used in 1925 by Otto Stern to verify Maxwell's distribution of molecular speeds. A beam of Bi_2 molecules (atomic mass of Bi $m = 208.98040\,\mathrm{u}$, $\mathrm{u} = 1.660539 \times 10^{27}\,\mathrm{kg}$) emitted from an oven at 850 K. The beam defined by slit S_1 was admitted into the interior of a rotating drum via slit S_2 in the drum wall. The identical bunches of molecules thus formed struck and adhered to a curved glass plate fixed to the interior drum wall, the fastest molecules striking near A, which was opposite to S_2, the slowest near B, and the others in between depending on their speeds. The density of the molecular deposits along the glass plate was measured with a densitometer. The density (proportional to the number of molecules) plotted against distance along the glass plate (dependent on v) made possible determination of the speed distribution. Assume that the drum is 10 cm in diameter and is rotating at 6250 rpm.

(a) Find the distance from A where molecules traveling at the most probable speed v_{m}, the average speed $\langle v \rangle$, and the rms speed v_{rms} will strike.

(b) The plot in (a) must be corrected slightly in order to be compared with Maxwell's distribution equation. Why?



Problem 4

In MRI (magnetic resonance imaging), a medical imaging technique used in hospitals to visualize internal structures of the body in detail, proton nuclear magnetization of water molecules is detected. In the absence of an external magnetic field, the proton spins are randomly oriented, resulting in zero macroscopic magnetization. When a magnetic field $\mathbf{B} = (0, 0, 7)$ Tesla is applied, proton spin angular momentum $\mathbf{S}$ (and magnetic moment $\boldsymbol{\mu}$) align with the field, splitting energy into two levels (spin quantum number $s = 1/2$). For a sample containing $9\,\mathrm{g}$ of H_2O at 300 Kelvin:

(a) Calculate the number of protons in the spin-up state (magnetic quantum number $m = 1/2$) and in the spin-down state ($m = -1/2$).

(b) Calculate the total magnetization (i.e. the net magnetic moment) due to protons in the sample, which is the source of the MRI signal.

Data for Problem 4

$$\boldsymbol{\mu} = \gamma \mathbf{S}$$

$$U = -\boldsymbol{\mu}\cdot\mathbf{B} = -\gamma B S_z = -m\hbar\gamma B, \quad m = \pm\tfrac{1}{2}$$

$$\gamma = 2.675222005 \times 10^{8}\,\mathrm{rad\,s^{-1}T^{-1}}$$

$$N_{\mathrm{A}} = 6.0221415 \times 10^{23}\,/\mathrm{mol}$$

$$\hbar = 1.054572 \times 10^{-34}\,\mathrm{J\cdot s}$$

$$k = 1.380650 \times 10^{-23}\,\mathrm{J/K}$$

Problem 5

A simple system is described by an alternative to the van der Waals equation of state, namely the first Dieterici equation

$$P = \frac{RT}{v-b}\exp\Big(-\frac{a}{RTv}\Big),$$

together with the Maxwell construction.

(a) Describe in a few sentences and one or two simple sketches what the Maxwell construction is.

(b) Which equations need to be solved to determine the critical point?

(c) Solve for the critical parameters v_c, T_c, and P_c.

(d) Calculate the thermal expansion coefficient α by solving $dP = 0$.

(e) Derive the relation $T\alpha = 1$ determining the inversion curve.

(f) Solve this simple equation, giving T as a function of v.

(g) Determine the inversion temperature T_i, i.e. the maximum value of T on this inversion curve, usually occurring in the limit $v \to \infty$.

Problem 6

Calculate the free energy per spin of a paramagnet with interaction energy

$$\mathcal{H} = -B\sum_{j=1}^{\tilde{N}} \sigma_j,$$

with $\sigma_j = \pm 1$. As a function of the scaled magnetic field B is this a convex or concave function? Calculate the total magnetization M and its fluctuation ΔM^2,

$$M \equiv \Big\langle \sum \sigma_j \Big\rangle, \qquad \Delta M^2 \equiv \Big\langle \Big(\sum \sigma_j \, - \, M\Big)^2 \Big\rangle,$$

as functions of $\tilde{N}$ and discuss on the basis of this how fluctuations can be ignored in the thermodynamic limit.