Department of Physics

Preliminary Exam: January 6–10, 2014

Day 2: Electricity, Magnetism and Optics

Tuesday, January 7, 2014

9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions.

2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.

3. The time allowed for this exam is three hours. All questions carry the same amount of credit. Manage your time carefully.

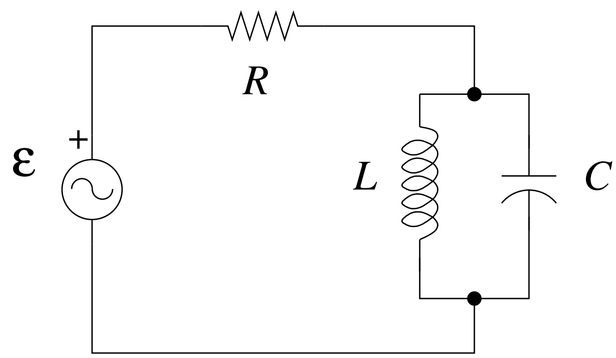
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.

5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.

6. The use of electronic calculators is permissible and may be needed for some problems. No other electronic device is permitted. Obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Attempt **all six** *problems. Each problem is worth 20 points.*

Problem 1



In the circuit above an ac voltage source $\varepsilon = V_0 \sin \omega t$ is connected in series with a resistor (resistance R) and a parallel inductor (inductance L)-capacitor (capacitance C) network.

(a) Determine the current in the:

i. resistor;

ii. capacitor;

iii. inductor.

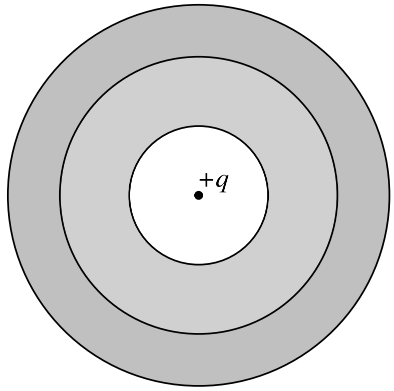
(b) Derive expressions for:

i. the *phase difference* between the voltage and the current in the resistor;

ii. the *phase difference* between the currents in the inductor and capacitor.

(c) Write down an expression for the *resonant frequency* and briefly discuss what happens in the circuit at this frequency.

Problem 2



A pair of snuggly fitting, hollow, dielectric spheres surround a point-charge q concentrically, as shown. The dielectric constants of the inner and outer dielectric layers are κ_1 and κ_2, respectively, and the radii of the dielectric boundaries are (from smallest to largest) a, b, and c.

Fields: Determine the fields ($\mathbf{D}$, $\mathbf{E}$, $\mathbf{P}$) in all regions. Here, $\mathbf{D}$ is the electric displacement, $\mathbf{E}$ is the electric field and $\mathbf{P}$ is the electric polarization.

Bound Charge: Determine all bound charge: surface, and bulk. (HINT: These relate to the $\mathbf{P}$ field.)

Energy: The charge is now removed (to infinity). What is the contribution, to the total work done, by the outer dielectric layer?

Problem 3

A long straight cylinder, $L \gg \rho$ (where ρ and L are the radius and length respectively) has a uniform magnetization of $\mathbf{M}$ along the axis of the cylinder (the z-axis). The cylinder is placed such that one of its ends is in contact with a flat *infinitely permeable* surface that makes up the x-y plane. (Note that the magnetic field $\mathbf{H}$ is always *perpendicular* to an infinitely permeable flat surface.)

(a) What property must the magnetic scalar potential possess at the surface?

(b) Use your answer to part (a) to show that when $L \gg \rho$ the cylinder sticks to the surface with a force given by

$$F \approx \frac{\mu_0}{2}\pi\rho^2 M^2 \quad .$$

Useful information:

- The force on an object with magnetization $\mathbf{M}$ in an external field $\mathbf{B}_{\text{ext}}$ is

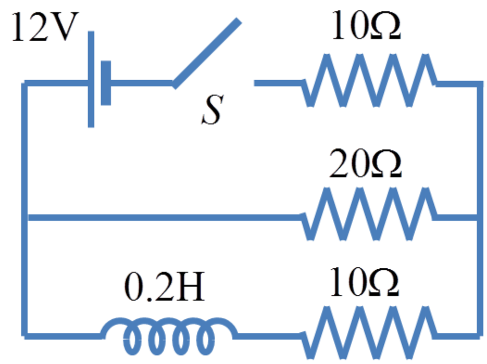
$$\mathbf{F} = -\int_V (\nabla \cdot \mathbf{M})\, \mathbf{B}_{\text{ext}}\, dv + \oint_S (\hat{\mathbf{n}} \cdot \mathbf{M})\, \mathbf{B}_{\text{ext}}\, da$$

- The magnetic induction on the axis of a cylinder of length L and radius ρ with a uniform magnetization $\mathbf{M} = M\hat{\mathbf{k}}$ and orientated so that its longitudinal axis is centered at the origin and pointing in the z-direction is

$$\mathbf{B}(z) = -\frac{\mu_0 M}{2}\left[\frac{z - \frac{L}{2}}{\sqrt{\rho^2 + (z - \frac{L}{2})^2}} - \frac{z + \frac{L}{2}}{\sqrt{\rho^2 + (z + \frac{L}{2})^2}}\right]\hat{\mathbf{k}}$$

Problem 4

(a) Consider the circuit in the figure below.

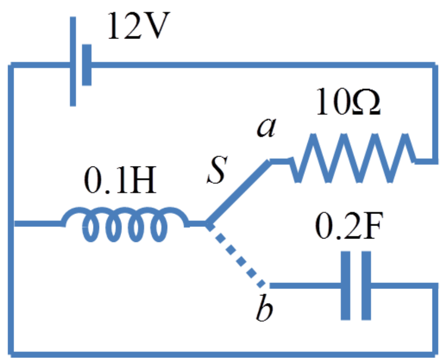


What is the current through each resistor:

i. just after the switch S is closed?

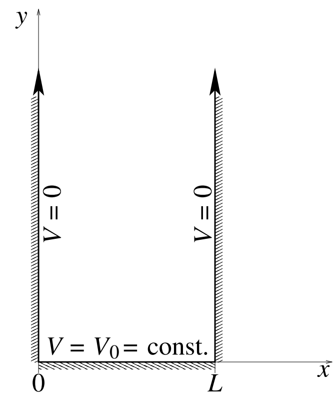
ii. after the switch has been closed for a long time?

(b) Now, consider this new circuit (see figure below):



After the switch S has been in position a for a long time, it is moved to position b. What is the maximum voltage across the capacitor?

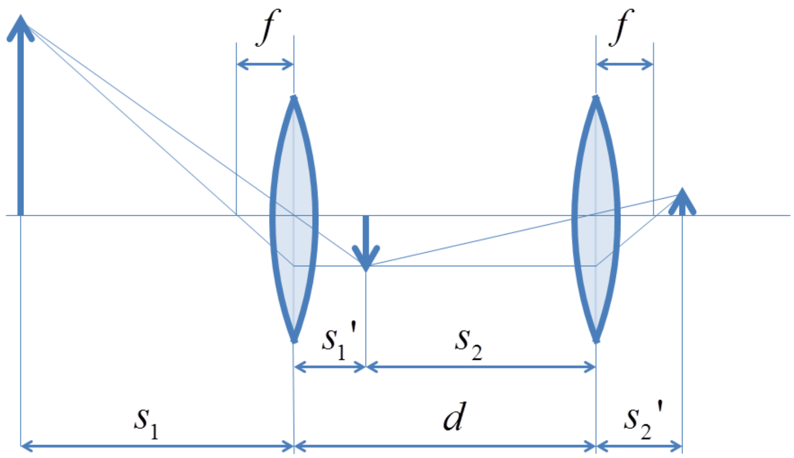
Problem 5



Two parallel, semi-infinite planar conductors are held at zero potential while a third plate, of width L, is held at a constant potential V_0, as shown.

Determine the potential, $\phi(x, y)$, in the region: $0 < x < L$ and $y > 0$.

Problem 6



An optical system consists of two *identical* lenses (each with focal length f) separated by a distance $d = 35\,\text{cm}$. An object is placed a distance $s_1 = 50\,\text{cm}$ to the left of the first lens (see figure above). The image produced by this system appears at a distance $s_2' = 18\,\text{cm}$ to the right of the other lens.

(a) What is the focal length f (in cm)?

(b) What is the total magnification of the system?