

Department of Physics
Preliminary Exam: January 6–10, 2014
Day 3: Quantum Mechanics and Modern Physics
Thursday, January 9, 2014
9:00 a.m.–12:00 p.m.

Instructions:

1. Write the answer to each question on a separate sheet of paper. If more than one sheet is required, staple all the pages corresponding to a *single* question together in the correct order. But, do *not* staple all problems together. This exam has *six* questions.
2. Be sure to write your exam identification number (*not* your name or student ID number!) and the problem number on each problem sheet.
3. The time allowed for this exam is three hours. *The amount of credit carried by each question is indicated against the question, and is not uniform.* Manage your time carefully.
4. If a question has more than one part, it may not always be necessary to successfully complete one part in order to do the other parts.
5. The exam will be evaluated, in part, by such things as the clarity and organization of your responses. It is a good idea to use short written explanatory statements between the lines of a derivation, for example. Be sure to substantiate any answer by calculations or arguments as appropriate. Be concise, explicit, and complete.
6. The use of electronic calculators is permissible and may be needed for some problems. No other electronic device is permitted. Obtaining preprogrammed information from programmable calculators or using any other reference material is strictly prohibited. The Oklahoma State University Policies and Procedures on Academic Integrity will be followed.

Attempt **all six** problems. Note that the problems have differing weights, as indicated against each problem.

Constants:

$$\begin{aligned}\hbar &= 1.0546 \times 10^{-34} \text{ J s} = 6.5821 \times 10^{-16} \text{ eV s}, \\ \mu_B &= 9.27 \times 10^{-24} \text{ J/T (Bohr magneton)}, \\ m_e &= 9.109 \times 10^{-31} \text{ kg}, \quad m_p = 1.673 \times 10^{-27} \text{ kg}.\end{aligned}$$

Problem 1

- (a) **10 pts.** In a Stern-Gerlach experiment, hydrogen atoms in their ground state move with speed $v_x = 14.5 \text{ km/s}$. The magnetic field is in the z direction and its maximum gradient is given by $dB_z/dz = 600 \text{ T/m}$. Ignore the spin of the proton.
- Find the maximum acceleration experienced by the hydrogen atoms.
 - How many lines would be expected on the detector plate?
 - If the region of the magnetic field extends over a distance of 75 cm and there is an additional 1.25 m from the edge of the field to the detector, find the maximum distance between every two lines on the detector plate.
- (b) **10 pts.** Five identical non-interacting particles are placed in a one-dimensional infinite square well of width $L = 1.0 \text{ nm}$. Compare the lowest total energy for the system if the particles are
- Electrons, which obey Fermi-Dirac statistics.
 - Pions, which have symmetric wave functions and whose mass is $264 m_e$.

Problem 2

- (a) **5 pts.** Consider a harmonic oscillator with frequency 1 Hz and mass equal to that of a proton. Find the root mean square displacement if the harmonic oscillator is in the ground state.
- (b) **15 pts.** Consider a hydrogen atom prepared in a superposition state

$$|\Psi(t=0)\rangle = \alpha|\Psi_{100}\rangle + \beta|\Psi_{210}\rangle .$$

- i. Write the wave function $|\Psi(t)\rangle$ at time t using the Hamiltonian of the hydrogen atom.
- ii. Use the wave function $|\Psi(t)\rangle$ to find the dipole moment defined by

$$\mathbf{p}(t) = \langle \Psi(t) | e\mathbf{r} | \Psi(t) \rangle .$$

Write the answer in terms of $\langle \Psi_{100} | z | \Psi_{210} \rangle$.

Problem 3

- (a) **10 pts.** Show that when $r \rightarrow 0$, a radial solution to the Schrödinger equation of a hydrogen-like atom,

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR(r)}{dr} \right) + \frac{2\mu}{\hbar^2} \left[E - \frac{kZe^2}{r} \right] R(r) = l(l+1) \frac{R(r)}{r^2},$$

is in the form $R(r) \propto r^l$. (HINT: Ignore the terms that become negligible relative to others as $r \rightarrow 0$.)

- (b) **10 pts.** The Hamiltonian for a spinless charged particle in a magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ is

$$H = \frac{1}{2m} \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right)^2$$

where e is the charge of the particle, $\mathbf{p} = (p_x, p_y, p_z)$ is the momentum, and $\mathbf{A}$ the vector potential.

- i. Write down the vector potential corresponding to a constant magnetic field in the z direction and the explicit form of the Hamiltonian in terms of B_z .
- ii. For the magnetic field described above, show that p_z is a constant of motion.

Problem 4

15 pts. Find eigenfunctions and eigenvalues of two interacting oscillators with Hamiltonian

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2 x_1^2 + \frac{1}{2}m\omega^2 x_2^2 + \frac{1}{2}v(x_1 - x_2)^2.$$

(HINT: Introduce new coordinates X_1, X_2 and their conjugate momenta P_1, P_2 to obtain the Hamiltonian in the form

$$H = \frac{P_1^2}{2m} + \frac{P_2^2}{2m} + \frac{1}{2}m\Omega_1^2 X_1^2 + \frac{1}{2}m\Omega_2^2 X_2^2.$$

Now the eigenfunctions and eigenvalues can be written down by inspection.) It is not required to derive the eigenfunctions of a single oscillator.

Problem 5

15 pts. For a free particle in a wave packet state

$$\psi(x) = \left(\frac{1}{\sqrt{2\pi}\sigma} \right)^{1/2} e^{-x^2/(4\sigma^2)},$$

- (a) Find the mean values: $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$, $\langle p^2 \rangle$, and $\langle px \rangle$.
 - (b) Find the uncertainty product $\Delta x \Delta p$.
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Problem 6

10 pts. Find when the matrix element of the angular momentum operator combination $(L_x L_y + L_y L_x)$ between two states with different quantum numbers is nonzero, i.e., the condition on the quantum numbers l', m' so that

$$\langle l, m | L_x L_y + L_y L_x | l', m' \rangle \neq 0.$$